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Subscription Models for Differential Access to Real-time Information

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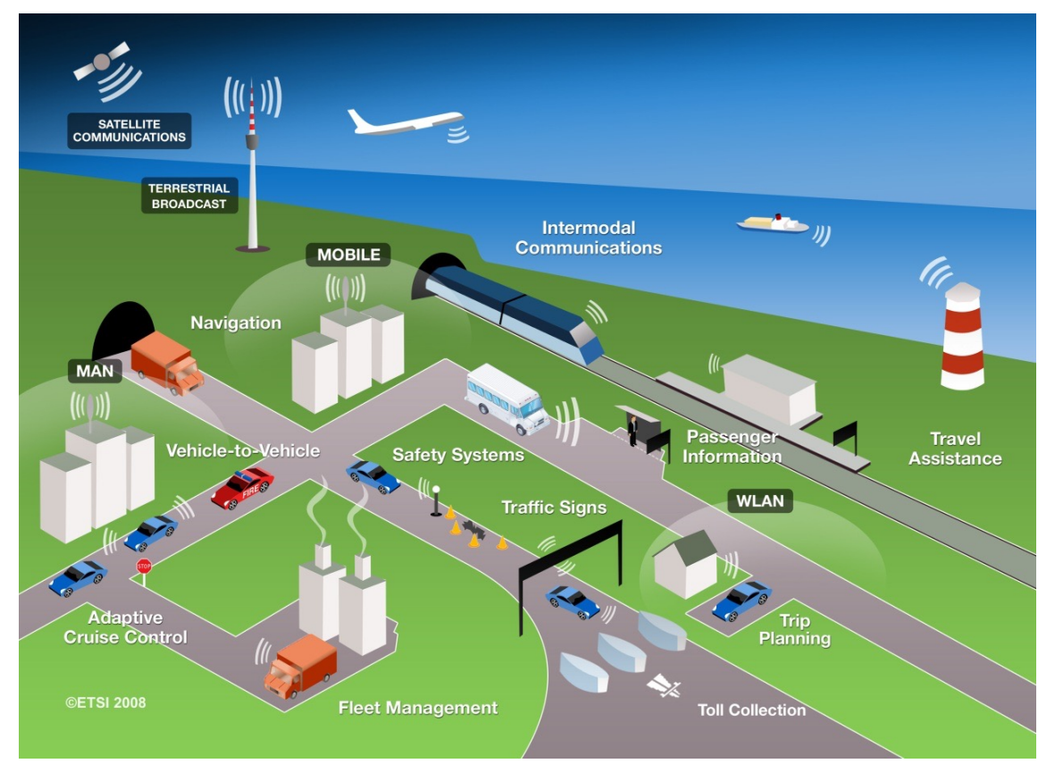
Overview

Background:

- Smart cities are built upon a **foundation of interconnected devices and technologies**, creating a networked infrastructure for **seamless data exchange**.
- Interconnected transportation systems generate data that provides insight into better decision making for both travelers and policy makers.
- Emerging questions: **how should information systems be designed?**
  - How much information should be revealed? At what times? At what cost?



Source: ETSI, 2008

- **Literature on models for real-time information:**
  - Emphasizes the role of advanced data processing for efficient city governance and citizen well-being within smart cities
  - Reference platforms like Wejo and INRIX use connected vehicle data to offer traffic updates and mobility insights.
  - Integrates the idea of **mobility as a marketplace** into the discussion, highlighting its potential in pricing shared information.

Research Questions

- How to determine what **percent of travelers would be willing to subscribe** for a given price of information subscription?
  - Are informed travelers always better off? And by how much?

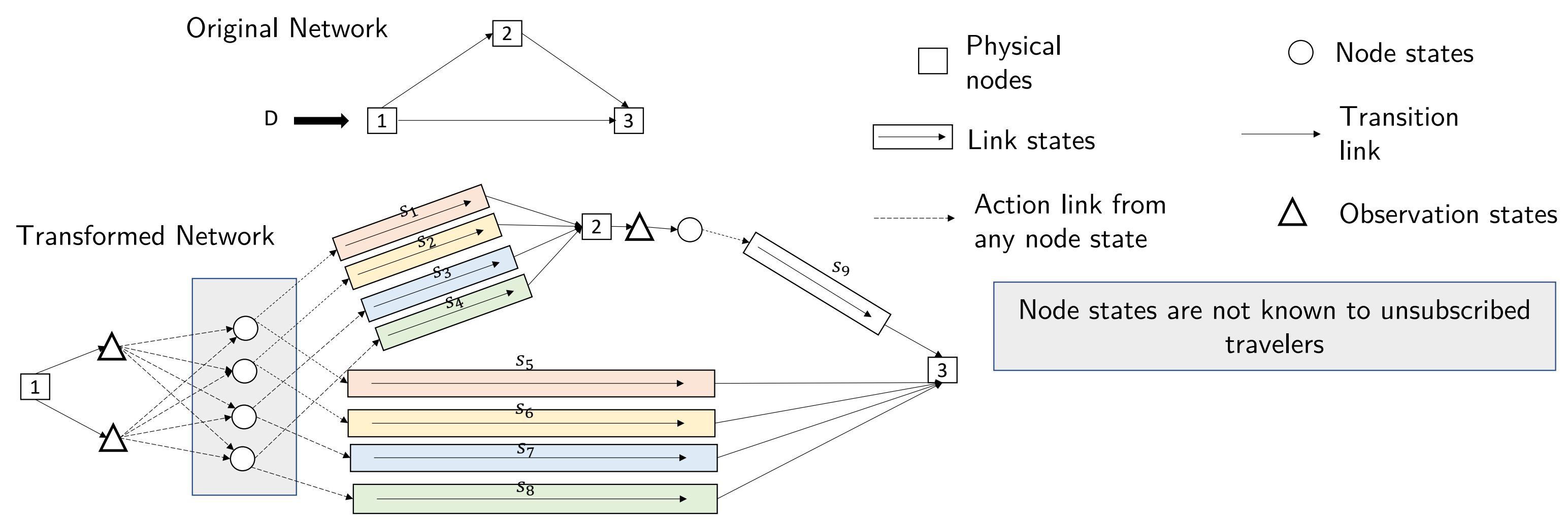
How to **assess the system level impact for different individual subscription rates** for available data and thus design prices that achieve broader agency objective?

- We study these problems through the **lens of network equilibrium** under the presence of travelers with and without information.
  - **Long-term impacts of information systems** in transportation systems have also been studied using various game-theoretic models: user equilibrium with recourse (UER) (Unnikrishnan et al., 2009; Rambha et al., 2018), Markovian congestion games (Calderone, 2017; Zimmermann et al., 2021) and Markovian traffic assignment (Oyama et al., 2022), policy-based dynamic traffic assignment (Gao, 2012), and Bayesian congestion games (Wu et al., 2021), applied across various applications like parking search (Boyles et al., 2015) and coordination in routing (Du et al., 2015).
  - The underlying tenet for these models **replaces the traveler's route choice with a policy or strategy** that adaptively determines the downstream arc at a node given the revealed information.
  - Efficient algorithms like hyperbush algorithms (Xu et al., 2022) can solve for strategy-based equilibrium flow efficiently on city-wide networks.

Motivating Example

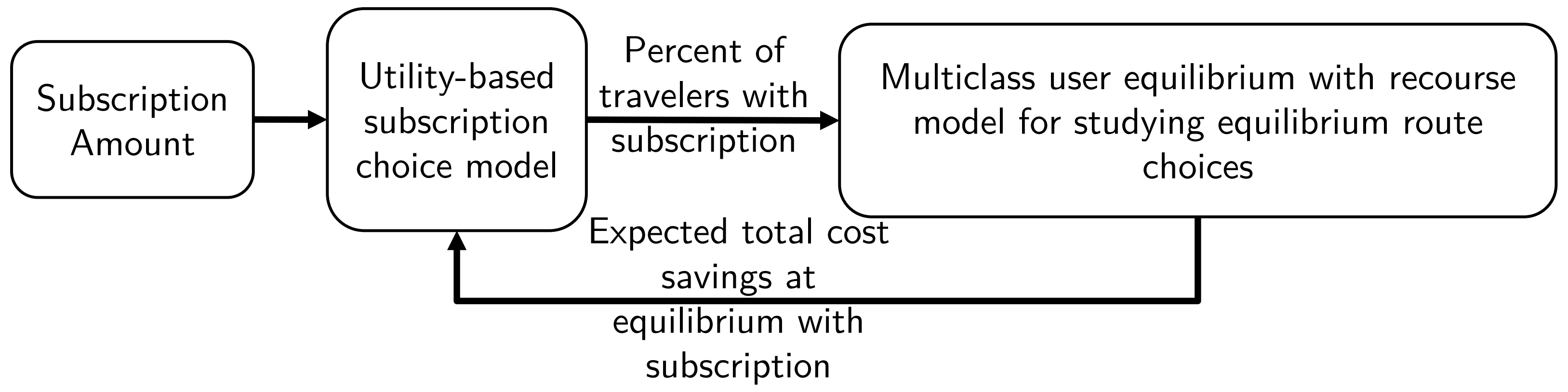
- Consider that there are two routes and upon arrival at 1, we can observe system in the states shown in the Table:
- The representation describe the states and probabilities associated with different nodes and links for both informed and uninformed travelers.
- The informed travelers have access to more information states compared to the uninformed travelers.

State of system	Tolled Link (1,2) maintained at free flow	Link (1,3) with no toll
θ_1	Toll = \$3	Capacity = 300
θ_2	Toll = \$3	Capacity = 600
θ_3	Toll = \$1	Capacity = 900
θ_4	Toll = \$1	Capacity = 1200



Node states are not known to unsubscribed travelers

Problem Formulation / Creating the networks



Subscription Amount

Utility-based subscription choice model

Percent of travelers with subscription

Multiclass user equilibrium with recourse model for studying equilibrium route choices

Expected total cost savings at equilibrium with subscription

- The objective of this problem formulation is to find the equilibrium choices given the subscription price and iteratively solve for the steady-state subscription prices.
- Similar to the UER formulation, we define link-states and node-states, with cost functions defined for each link-state.
- The shortest expected cost policy can be formulated as a Markov decision process, and optimal policy represented using sparse matrices

Link-state cost ($\boldsymbol{c}$) → Node costs ($\boldsymbol{C}^{\pi}$)
Policy flows ($\mathbf{y}^{\pi}$) → Link-state flows ($\mathbf{x}^{\pi}$)

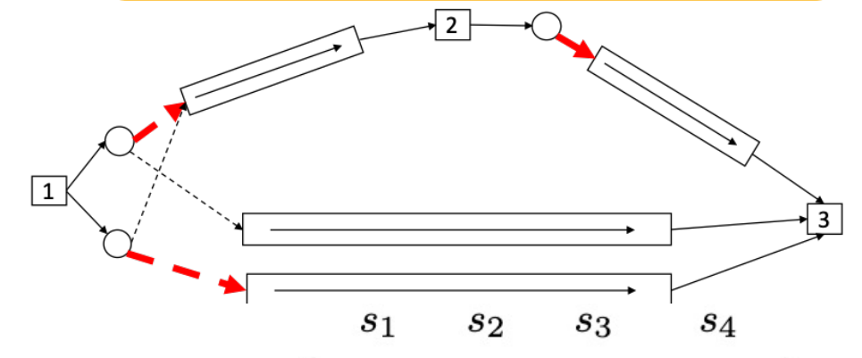
$$C^{\pi} = F_{\pi}\mathbf{c} \qquad \mathbf{x}^{\pi} = F_{\pi}^{\top}\mathbf{y}^{\pi}$$

MUER condition

$$y_u^{\pi} > 0 \Rightarrow C_u^{\pi}(\mathbf{y}) = \min_{\bar{\pi}\in\hat{\Pi}_{\Uparrow}^{v}} C_u^{\bar{\pi}}(\mathbf{y}) \quad \forall v \in Z, \pi \in \hat{\Pi}_{\Uparrow}^{v}, \text{ and}$$

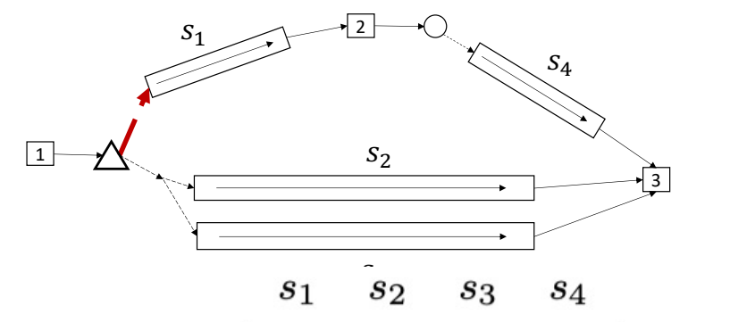
$$y_u^{\pi} > 0 \Rightarrow C_u^{\pi}(\mathbf{y}) = \min_{\bar{\pi}\in\hat{\Pi}_{\Downarrow}^{v}} C_u^{\bar{\pi}}(\mathbf{y}) \quad \forall v \in Z, \pi \in \hat{\Pi}_{\Downarrow}^{v}.$$

Subscribed traveler Policy selecting link (1,3) only if it is in s_3



$$F_{\pi_2} = \begin{array}{cc} & \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0.6 & 0 & 0.4 & 0.6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Unsubscribed traveler Policy selecting link (1,2)



$$F_{\pi_3} = \begin{array}{cc} & \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Algorithm 1 Iterative algorithm for solving the subscription model for a given subscription rate β

```
Step 1: Start with a guess $p \in (0,1)$
Step 2: Solve MUER using Algorithm 2
Step 3: Compute $C_{savings}(p)$ and updated value of proportion using Equation (12). If the percent change is below a set threshold of 0.0001, stop. Else, go back to Step 2.
```

For a given percent of subscribed travelers (p), we determine the $C_{savings}^{uv}(p)$ = cost difference between optimal policies for subscribed and unsubscribed travelers at equilibrium for an OD pair

We assume fixed price for information subscription (β)

- More general pricing models are possible such as pricing by OD pair.

Deterministic choice model:
If $\beta > C_{savings}^{uv}(p)$, then a traveler chooses not to subscribe

Stochastic choice model
Proportion choosing to subscribe is given by a binary logit model with scale parameter $\phi \approx 1$

$$p = \frac{1}{1 + e^{\beta - C_{savings}(p)}}$$

Algorithm 2 Policy-based gradient projection algorithm for solving UER

```
Initialize $\hat{\Pi}_{uv}$ = NULL for all $(u,v) \in Z^2$. Set iteration number $n = 0$. Set $x_s\rfloor_n = 0$ for all $s \in S$. Set $c_s = c_s(0)$ for all $s \in S$. Set GAP $\leftarrow \infty$
while GAP $> \varepsilon$ do
    for Each destination $v \in Z$ and for each class of subscribed and unsubscribed travelers do
        Find shortest policy $\pi_v^*$ towards destination $v$ using TD-OSP algorithm in (30)
        for Each origin $u \in Z$ with $d_{uv} > 0$ do
            Shortest policy $\pi_{uv}^* \leftarrow \pi_v^*$
            if $\pi_v^* \notin \hat{\Pi}_{uv}$ then $\hat{\Pi}_{uv} \leftarrow \hat{\Pi}_{uv} \cup \{\pi_v^*\}$
            if $|\hat{\Pi}_{uv}| = 1$ then
                Set $y_u^{\pi_{uv}^*}\rfloor_{n+1} \leftarrow d_{uv}$
            else $|\hat{\Pi}_{uv}| > 1$
                Set total flow to shift towards basic policy as zero: $\Delta y^* \leftarrow 0$
                for $\pi \in \hat{\Pi}_{uv}$ such that $\pi \neq \pi_{uv}^*$ do
                    $\Delta y \leftarrow \min\left\{ y_u^{\pi}\rfloor_n, \frac{C_u^{\pi} - C_u^{\pi_{uv}^*}}{\sum_{s\in S} c_s'(x_s\rfloor_n)(f_{\pi}(u,s) - f_{\pi_{uv}^*}(u,s))^2} \right\}$
                    $y_u^{\pi}\rfloor_{n+1} \leftarrow y_u^{\pi}\rfloor_n - \Delta y$
                    $\Delta y^* \leftarrow \Delta y^* + \Delta y$
                Set $y_u^{\pi_{uv}^*}\rfloor_{n+1} \leftarrow y_u^{\pi_{uv}^*}\rfloor_n + \Delta y^*$
    GAP $\leftarrow (\sum_{s\in S} c_s x_s\rfloor_n) \left(\sum_{(u,v)\in Z^2} d_{uv} C_u^{\pi_{uv}^*}\right)^{-1} - 1$
    Update $x_s\rfloor_{n+1}$ using $y_u^{\pi}\rfloor_{n+1}$ values
    $n = n + 1$
```

Gradient Projection Algorithm with Sparse-Matrix Representation

Proposition:. At equilibrium, the total link-state flows are unique while the flows for unsubscribed and subscribed travelers are not unique.

Proposition: At MUER solution, $C_{savings}(p) \geq 0$ for all $p \in [0,1]$.That is, costs for subscribed travelers is always less than or equal to the costs for unsubscribed travelers.

Proposition: Unique fixed point
Broadly, for a given β, we have the following cyclic relation: $\boldsymbol{p} = \mathcal{C}(\mathcal{E}(\boldsymbol{p}), \boldsymbol{\beta})$
Both subscription choice model ($\mathcal{C}$) and user-equilibrium model ($\mathcal{E}$) are continuous functions, and $p \in [0,1]$ is a compact convex set, using Brouwer's fixed point theorem, there exist a fixed point.

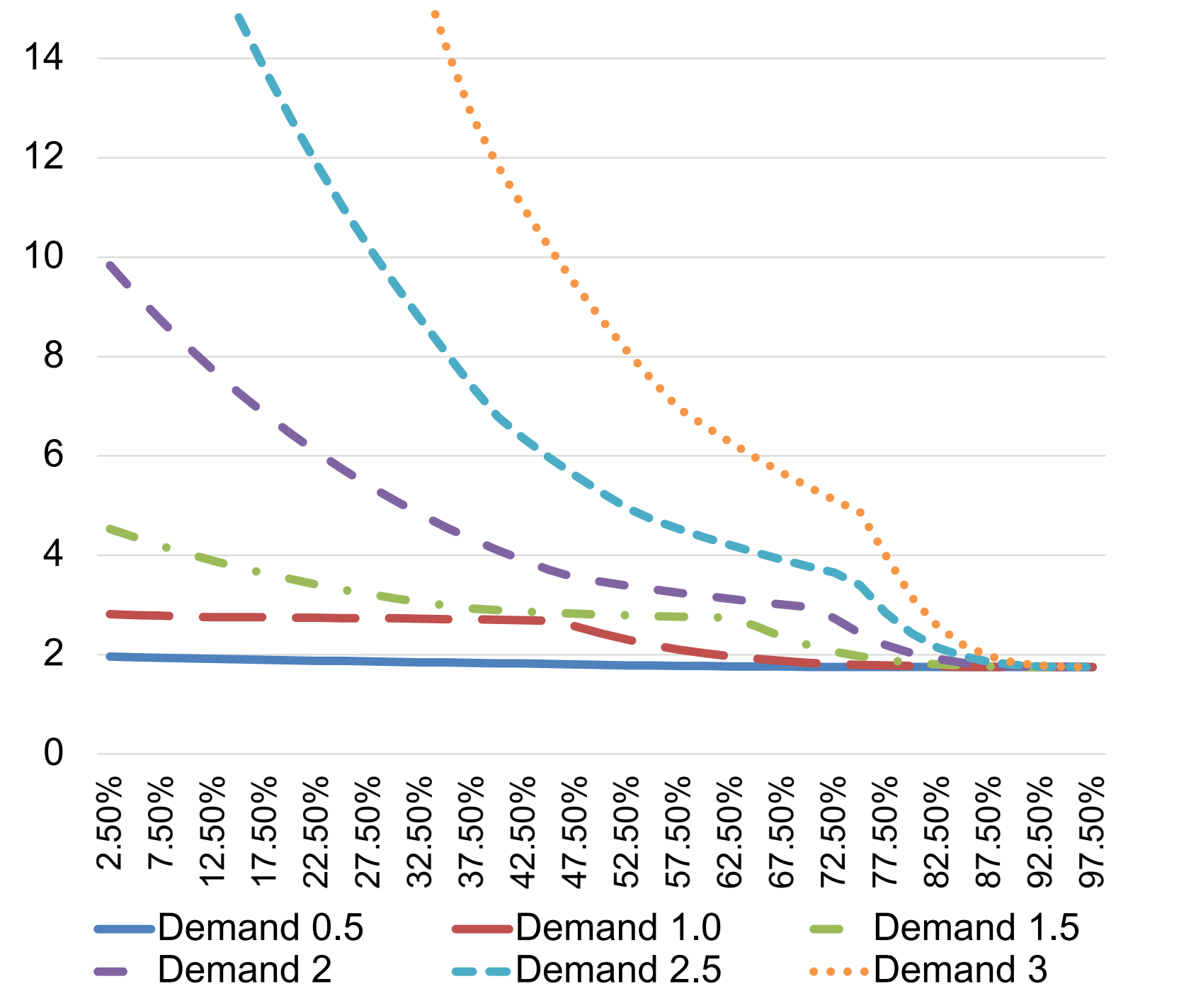
Experimental Findings on Test Networks

We tested our analysis on acyclic networks (Three-node network, Braess network, and Nguyen-Dupuis Network) with introduced supply-side uncertainties.

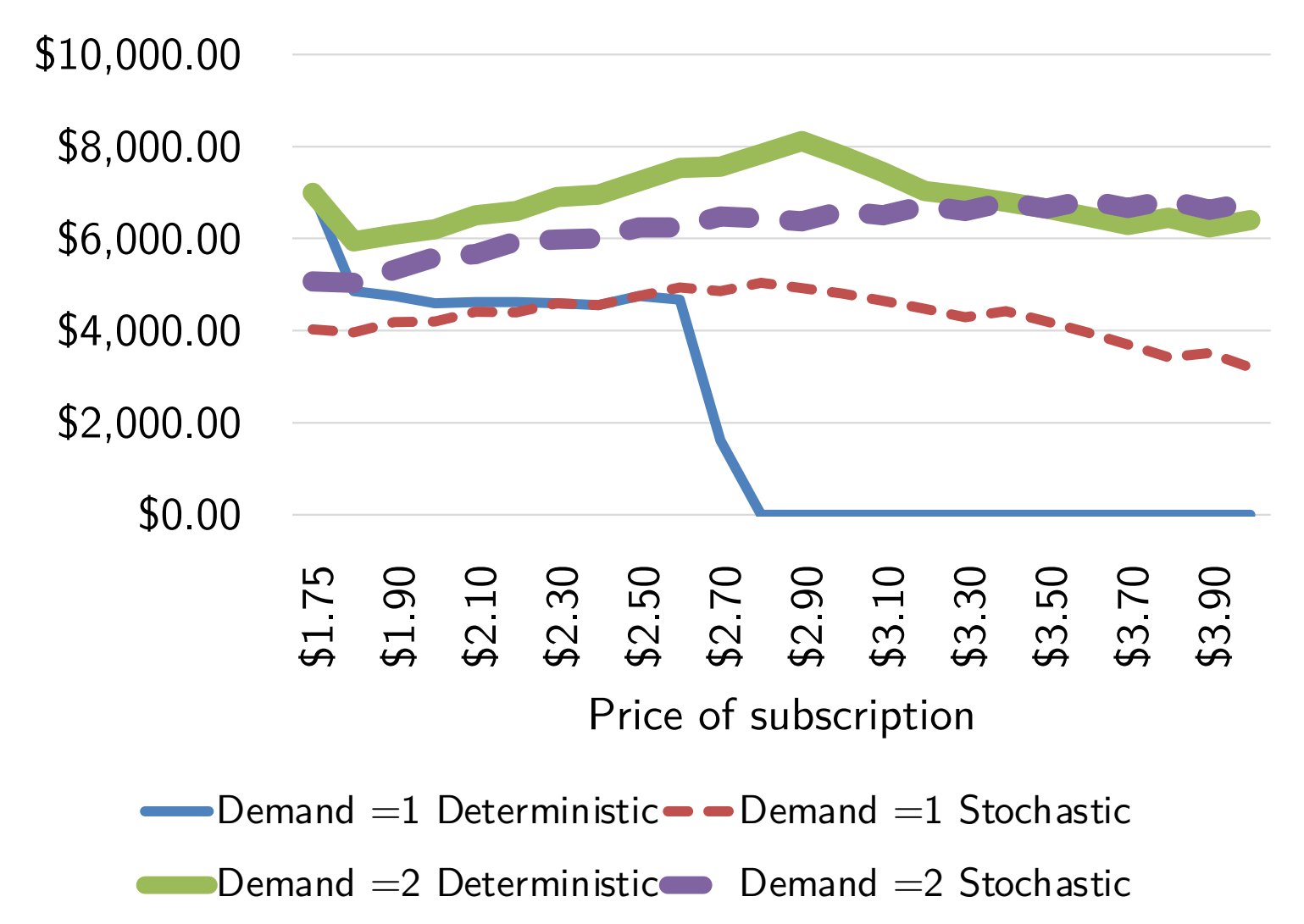
Findings include:

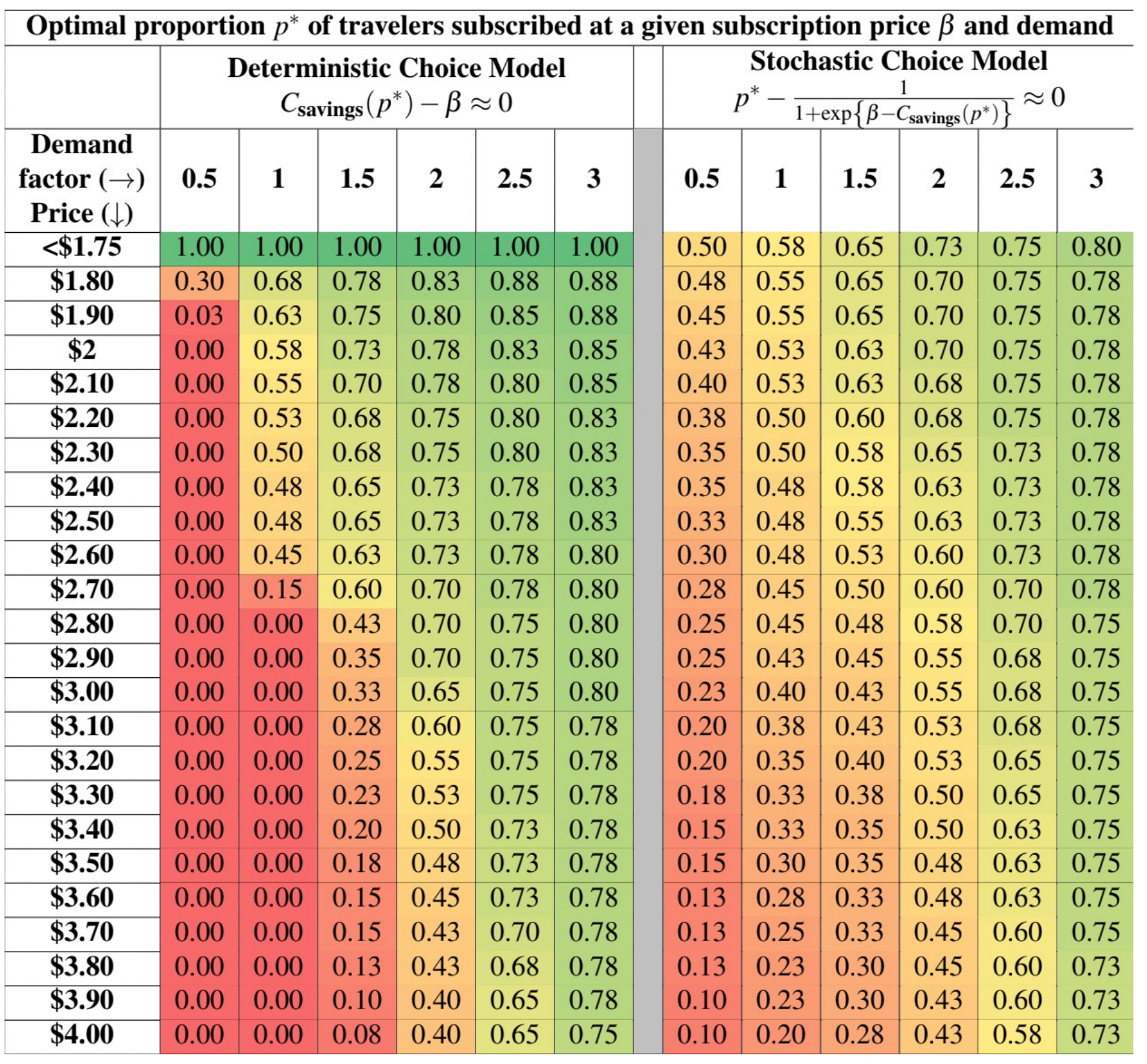
- Uninformed travelers have higher costs due to limited number of policies
- The cost savings reduce as more travelers continue to subscribe. That is,
$$\frac{\partial C_{Savings}}{\partial p} \leq 0$$
for all proportional splits
- Typically, overall system cost is minimized when more travelers opt to subscribe; however, for networks with Information Paradox (Wijayaratna et al. (2017)), the **total system cost may increase if more travelers subscribe** beyond a threshold.
- As prices increase, a smaller proportion of travelers subscribe. Similarly, **as demand increases, the percent of subscribed travelers for a given price increases** since higher demand results in higher congestion and thus there is more benefit in subscribing to information
- If the agency cares about maximizing revenue in contrast to minimizing TSTC, the optimal prices are different.

Cost difference (\$) with increasing demand as a function of the proportion of subscribed travelers



Obtained Revenue (\$) from charging a specific price of subscription





Optimal proportion p^* of travelers subscribed at a given subscription price β and demand

	Deterministic Choice Model $C_{savings}(p^*) - \beta \approx 0$							Stochastic Choice Model $p^* - \frac{1}{1+\exp\{\beta - C_{savings}(p^*)\}} \approx 0$					
Demand factor (→) Price (↓)	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**		**0.5**	**1**	**1.5**	**2**	**2.5**	**3**
<\$1.75	1.00	1.00	1.00	1.00	1.00	1.00		0.50	0.58	0.65	0.73	0.75	0.80
\$1.80	0.30	0.68	0.78	0.83	0.88	0.88		0.48	0.55	0.65	0.70	0.75	0.78
\$1.90	0.03	0.63	0.75	0.80	0.85	0.88		0.45	0.55	0.65	0.70	0.75	0.78
\$2	0.00	0.58	0.73	0.78	0.83	0.85		0.43	0.53	0.63	0.70	0.75	0.78
\$2.10	0.00	0.55	0.70	0.78	0.80	0.85		0.40	0.53	0.63	0.68	0.75	0.78
\$2.20	0.00	0.53	0.68	0.75	0.80	0.83		0.38	0.50	0.60	0.68	0.75	0.78
\$2.30	0.00	0.50	0.68	0.75	0.80	0.83		0.35	0.50	0.58	0.65	0.73	0.78
\$2.40	0.00	0.48	0.65	0.73	0.78	0.83		0.35	0.48	0.58	0.63	0.73	0.78
\$2.50	0.00	0.48	0.65	0.73	0.78	0.83		0.33	0.48	0.55	0.63	0.73	0.78
\$2.60	0.00	0.45	0.63	0.73	0.78	0.80		0.30	0.48	0.53	0.60	0.73	0.78
\$2.70	0.00	0.15	0.60	0.70	0.78	0.80		0.28	0.45	0.50	0.60	0.70	0.78
\$2.80	0.00	0.00	0.43	0.70	0.75	0.80		0.25	0.45	0.48	0.58	0.70	0.75
\$2.90	0.00	0.00	0.35	0.70	0.75	0.80		0.25	0.43	0.45	0.55	0.68	0.75
\$3.00	0.00	0.00	0.33	0.65	0.75	0.80		0.23	0.40	0.43	0.55	0.68	0.75
\$3.10	0.00	0.00	0.28	0.60	0.75	0.78		0.20	0.38	0.43	0.53	0.68	0.75
\$3.20	0.00	0.00	0.25	0.55	0.75	0.78		0.20	0.35	0.40	0.53	0.65	0.75
\$3.30	0.00	0.00	0.23	0.53	0.75	0.78		0.18	0.33	0.38	0.50	0.65	0.75
\$3.40	0.00	0.00	0.20	0.50	0.73	0.78		0.15	0.33	0.35	0.50	0.63	0.75
\$3.50	0.00	0.00	0.18	0.48	0.73	0.78		0.15	0.30	0.35	0.48	0.63	0.75
\$3.60	0.00	0.00	0.15	0.45	0.73	0.78		0.13	0.28	0.33	0.48	0.63	0.75
\$3.70	0.00	0.00	0.15	0.43	0.70	0.78		0.13	0.25	0.33	0.45	0.60	0.75
\$3.80	0.00	0.00	0.13	0.43	0.68	0.78		0.13	0.23	0.30	0.45	0.60	0.73
\$3.90	0.00	0.00	0.10	0.40	0.65	0.78		0.10	0.23	0.30	0.43	0.60	0.73
\$4.00	0.00	0.00	0.08	0.40	0.65	0.75		0.10	0.20	0.28	0.43	0.58	0.73

Conclusions and Ongoing Work

- **Conclusions**:
  - Proposed a convex program to achieve the joint equilibrium of unsubscribed and subscribed travelers. The framework can assist with assessing the impact of pricing the information towards a future of smart cities.
- **Ongoing work**:
  - Deriving the properties of derivatives
  - Solving the information design problem: What price and what real-time information can enable lowest TSTC
  - Studying the equity and welfare impacts of differential information access
  - Integrate tests on large-scale networks

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