Safety, Liability, and Insurance Markets in the Age of Automated Driving

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Abstract

This paper investigates two fundamental questions related to safety and insurance in the age of automation. First, we touch upon the question of safety and liability under infrastructure-assisted automated driving. In such an environment, automakers provide vehicle automation technology while infrastructure service providers (ISSPs) provide smart infrastructure services. Additionally, customers can receive coverage for accidents from either of these actors but also from legacy auto insurers. We investigate the effect of market structure on safety and accident coverage and show that an integrated monopoly provides full coverage and fully accounts for accident costs when choosing safety levels. However, in the Nash setting, even though full coverage obtains, lack of coordination leads to partial internalization of accident costs by the automaker. Moreover, multiple equilibria might exist, some of them undesirable. We show that, both in the presence and absence of legacy insurance, an appropriate liability rule can induce optimal safety levels under the Nash setting. Our second question concerns itself with the role of legacy auto insurance in the age of infrastructure-assisted automated driving. Our analysis shows that the industry is not necessary for optimal coverage when the cost of accidents is known in advance and all possible accident scenarios are contractible. In fact, their presence can even harm safety, even though it ensures full coverage for accidents. However, when only insurance contracts with capped liability for automakers and ISSPs are available, we cannot rule out that customers benefit and purchase insurance from legacy insurers. Thus, the disappearance of the industry in the age of automated driving is not a foregone conclusion.

Keywords - Driving automation, automated vehicles, infrastructure-assisted automated driving, safety, insurance

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1 Introduction

Over the past decade, the development of automated vehicles has generated a lot of interest and has provided a glimpse into an exciting future. In an environment with fully automated vehicles,

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travel will become more convenient, accessible, and time-effective. However, despite tremendous progress (e.g.: Level 2 and Level 3 vehicles (Tucker, Sean, 2022; Communications, 2022), driverless taxi services (Elias, 2022)), several challenges (e.g.: weather-related issues; infrastructure changes; interactions with human-driven vehicles) remain to achieve that vision. To address these challenges, researchers, public agencies (FHWA, 2021), and private investors (Grinell, 2020; Hertz, 2022) have considered investing in infrastructure to assist the driving task. Such efforts in digitalization–the equipping of infrastructure to support automated driving–could lead to what we termed, in a recent article, vehicle-infrastructure cooperation (Vignon et al., 2022). In this vision, "*both the level of automation in the future fleet and the level of digitalization in the future infrastructure will be heterogeneous*" (Vignon et al., 2022). We analyzed such a paradigm and showed that a) it can be socially efficient due to the heterogeneous nature of travel decisions and b) cooperation between automakers and infrastructure support service providers (ISSPs) is preferable to competition between these two entities.

This paper further investigates vehicle-infrastructure cooperation from the perspective of safety, liability, and insurance. Indeed, one of the key benefits that will accrue to customers through automation and digitalization is safety. For example, using National Highway Traffic Safety Administration (NHTSA) data, Fagnant and Kockelman (2015) identify that 90 % of traffic accidents involve human error. When considering the substantial costs of crashes to society–$226 billion in 2005 (2021 dollars) according to Cambridge Systematics (2008)–improved safety resulting from vehicle automation would generate enormous savings to customers and society in general. Moreover, most of the remaining challenges in developing automated vehicles revolve around safety–as discussed in the introduction to Vignon et al. (2022)–and research shows that customers have high expectations for automated vehicle safety (Shariff et al., 2021; Shariff, 2021). Importantly, after a series of road incidents involving automated vehicles, regulators are also starting to show concerns about automation technology's safety and readiness for commercialization (Elliott, 2021). Thus, it appears that finding cost effective means to address the remaining safety concerns of automated vehicles is critical if automakers want to encourage adoption.

Additionally, though automated vehicles will increase safety, we are unlikely to see accident-free roads. Indeed, vehicles and infrastructure could still malfunction; pedestrians are still independent agents who can interfere with vehicle operation; and human-driven vehicles (HDVs) will co-exist for a long period of time with AVs, potentially leading to more crashes than in an AV-free environment since HDVs might reduce their level of care (Chatterjee and Davis, 2013; Elvik, 2014; Di et al., 2020). In this context, one might ask who should bear the costs for accidents in which an AV is involved. Currently, under tort law, automakers are liable when negligence or a lack of reasonable care in the design and manufacturing of their vehicles is shown to be the cause of an accident. Otherwise, customers assume the driving risks: they are responsible for covering damages resulting from accidents in which they are involved. In order to be able to cover the potentially enormous costs, customers purchase insurance coverage. In exchange for regular premiums[1], they are able to obtain coverage for damages they sustain and for damages they inflict on others in the event of an accident.

In a fully automated driving environment, automakers and ISSPs not only provide driving technology, but are also responsible for the driving task. As such, our intuition suggests that they will assume responsibility for accidents that result from automated driving and therefore alter or possibly deal a fatal blow to the auto insurance industry[2]. However, despite the intu-

[1]Often paid monthly or biannually

[2]Or, at least, the part of the industry that caters to drivers. We do not here discuss insurance for automakers.

itiveness of such a future, its emergence is not a given for a number of reasons. First, in the current legal regime, such a future hinges on the willingness of automakers/ISSPs–rather than their being legally compelled to–to assume financial responsibility for accidents in which their technology is involved. While some automakers, like BMW, have decided to go that route (even below Level 5 (Tucker, Sean, 2022)), others such as Tesla, have been content to let current liability rules for drivers prevail and have, mostly, eschewed responsibility for accidents related to their technology[3] (Communications, 2022). Second, the current legal regime might simply be inadequate to deal with an environment dominated by automated driving. Indeed, in the current (mostly) negligence-based regime, customers must demonstrate, in court, that accidents could be attributed to automaker negligence. In an automated driving environment in which automakers are not automatically liable for accidents, collecting evidence and adjudicating such claims in a timely manner could demand enormous resources. Third, even when such resources are deployed, automated driving providers would only be held liable for accidents which could have been reasonably avoided. Thus, there might still exist a set of accidents whose economic burden customers must shoulder. In this context, auto insurers would still play a role. In this environment featuring customers, automakers, ISSPs, and legacy auto insurers, we naturally wonder how the legal environment should evolve and the subsequent repercussions for driving safety.

In this paper, we focus on answering the following questions relating to liability, safety, and insurance in an automated driving environment:

1. How does market structure affect safety and liability in an automated driving environment that features automakers, ISSPs, and insurance providers?

2. How should liability for accidents be apportioned among the different actors in this environment?

3. Does legacy auto insurance still have a role to play in the age of automated driving?

Our work is divided as follows. Section 2 presents the relevant literature and places the present work in its proper context. Section 3 introduces our model and its basic components, provides an analysis of equilibria in different settings, and discusses policy implications. Then, in Section 4, we relax the assumption of certainty in accident costs to understand how it affects the insights previously derived. Finally, Section 5 concludes the paper, and provides future research directions.

2 Literature review

The question of liability and automated driving has been explored in a number of works over the past decade.

First, researchers have explored and discussed ethical problems that arise in the design of AVs (Nyholm and Smids, 2016; Contissa et al., 2017; Thornton et al., 2017; Himmelreich, 2018; Nyholm, 2018; Borenstein et al., 2019; Wu, 2020; Feess and Muehlheusser, 2022). In this strand of literature, researchers discuss the potential moral dilemma that arises when vehicles must be pre-programmed to make decisions that might endanger certain lives in order to save others when a fatal accident is unavoidable.

[3] Sometimes rightly (Elliott and Felton, 2023).

Second, researchers have used economic and legal concepts to study how the legal environment should evolve with the advent of automated driving (Lohmann, 2016; Di et al., 2020; Shavell, 2020; Dawid and Muehlheusser, 2022; Feess and Muehlheusser, 2022; Dawid et al., 2023). These works study how different liability rules and regimes might affect the safety of automated vehicles; how they might favor the introduction and adoption of AVs; or how policies such as investment in smart infrastructure can shape the resulting equilibrium in the automobile market. Generally, in these works, a regulator or social planner decides upon a liability regime (e.g.: strict liability vs negligence-based liability, etc.) to minimize the social cost of driving. This liability regime, in turn, affects how much an AV manufacturer invests in the safety of their vehicles. The framework may include other features (such as mixed traffic or smart infrastructure provision) and their effect on the probability of accident. None of these works consider the question of insurance for automated driving customers and how it affects market outcomes but always assume that the liability regime is such that AV manufacturers bear full economic responsibility for accidents.

To incorporate the notion of liability rules and adequately answer their research questions, contributors to the aforementioned strand of literature draw heavily on the product liability literature (Posner, 1972; Oi, 1973; Ordover, 1979; Posner and Landes, 1980; Polinsky, 1980; Shavell, 1980; Landes and Posner, 1985; Daughety and Reinganum, 2013). Here, the focus is on the following basic problem: given information (or lack thereof) on product safety and risk preferences for both customers and product manufacturer, how do different liability regimes affect equilibrium outcomes (e.g.: safety levels, product demand and price, etc.)? Closely related to that literature is that of insurance. Indeed, depending on the liability regime, customers (and even firms) might purchase insurance services in order to cover accident related liability costs. Moreover, product manufacturers themselves might directly provide insurance. Most importantly, by considering the fact that insurance contracts are means through which an agent can offload a part or a totality of its liability burden on another agent, one could readily consider the question of product liability to be a question of insurance contract design between two parties. These contracts have been studied in detail over the years. Some articles have dealt with the issue of insurance in competitive markets (Ehrlich and Becker, 1972; Rothschild and Stiglitz, 1976; Cook and Graham, 1977; Schlesinger, 1983) and monopolistic markets (Stiglitz, 1977; Ligon and Thistle, 1996) under different information structures; others have looked at insurance under the threat of adverse selection and moral hazard and sought to evaluate their effects on insurance markets and insurance provision (Pauly, 1974; Dionne, 1982). Some have even tried to estimate the effect of connectivity on insurance cost, accounting for issues like privacy (Jin and Vasserman, 2021).

In the present work, we propose to investigate the question of liability in the age of automated driving by focusing on the effect of market structure on the provision of safety. By explicitly considering the fact that full coverage from automated driving providers is not a given, we derive insights as to the role that both an appropriate liability rule and auto insurance from legacy insurers can play in making automated driving safer and limiting economic losses from accidents. We draw heavily from our previous work (Vignon et al., 2022), from the insurance literature, and naturally, from the literature on liability in the age of automated driving. In our setting, an AV manufacturer and an ISSP provide automation and infrastructure services, respectively, to customers who care, among other things, about safety. AV manufacturer, ISSP, and legacy insurers can also provide insurance–implicitly or explicitly–to their customers. We first consider the case in which accident costs are known ahead of time and show that, in such a setting, legacy insurance is not relevant to customer's liability burden. In fact, absent regulation, the presence of legacy insurers might lead to worse safety levels. Then, we consider the case in which

Notation	Description
z^M	Vehicle automation quality
z^I	Infrastructure automation quality
$\phi_1^{j,n}$	Premium for coverage on road j demanded by agent $n \in \{M, I, L\}$
$\phi_2^{j,n}$	Coverage amount for accident on road $j \in \{r, s\}$ by agent $n \in \{M, I, L\}$
c^n	Investment cost for $n \in \{M, I\}$
τ^n	Price of technology sold by agent $n \in \{M, I, L\}$
V	Value of car ownership
λ	Total demand for vehicle ownership
p^j	Crash probability per mile driven on road $j \in \{r, s\}$
$\tilde{p}^k$	Probability of state $k \in \{0, r, s\}$
W^k	Wealth in state $k \in \{0, r, s\}$
$U(\cdot)$	Utility function
$P(\cdot, \cdot)$	Accident probability function
$C^n(\cdot)$	Per unit production cost of $n \in \{M, I\}$

(a) *Frequently used variables*

Notation	Description
η^j	Share of road $j \in \{r, s\}$
l	Total road length
$\tilde{W}^0$	Initial wealth level
d^j	Accident severity on road $j \in \{r, s\}$
P^0	Crash probability for human-driven vehicle
λ^0	Market size

(b) *Frequently used parameters*

Table 1: *Frequently used notations*

accident costs are uncertain and show that, in such a context, legacy insurers may improve market outcomes for consumers. However, numerical examples suggest that legacy insurance could also be harmful in that setting.

3 Known accident costs

3.1 Model

Consider a roadway used exclusively by fully AV owners. These owners purchase their vehicles from a vehicle manufacturer who decides the quality of the technology (e.g.: sensors, algorithms, other hardware and software components, etc.) with which to equip them. The manufacturer's quality choices affect the crash probability of his vehicles and, therefore, his bottom line. Indeed, customers will shy away from an accident-prone product. We consider that all losses and accidents are the results of vehicle technology. Importantly, we do not consider owners' efforts in maintaining vehicles in fully functional order. Thus, we do not account for moral hazard and/or adverse selection.

A portion of the roadway is managed by an ISSP. This ISSP installs and operates AV-related technology on the roadway. These upgrades contribute to reducing the crash probability on the roadway and are features that customers might use as complements or substitutes to automation technology.

AV owners face three mutually exclusive states of the world while using an AV as shown in Figure 1. In the first state, they travel without accident on the full length of the road (Figure 1-a). In the second state, they are involved in an accident on the regular (r) portion of the road and face potential losses as a result (Figure 1-b). In the third state, they are instead involved in an accident on the smart (s) portion of the road–but not on the regular portion (Figure 1-c). To cover their potential losses, customers have the option of obtaining insurance from three parties: the manufacturer (M), the ISSP (I), and a legacy auto insurer (L), each of whom provides insurance coverage in exchange for a premium.

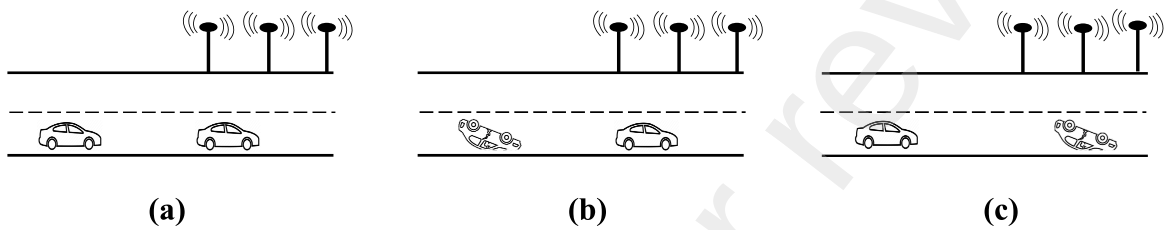

Figure 1: *Schematic of state (a) no accident, (b) accident on regular portion, and (c) accident on smart portion of the road.*

Formally, then, user preferences can be described by the following equations:

$$V = \tilde{p}^0 \cdot U(W^0) + \tilde{p}^r \cdot U(W^r) + \tilde{p}^s \cdot U(W^s)$$

$$W^k = \begin{cases} \tilde{W}^0 - \sum_{n \in \{M,I,L\}} (\tau^n + \sum_{j \in \{r,s\}} \phi_1^{j,n}) & \text{if } k = 0 \\ \tilde{W}^0 - \sum_{n \in \{M,I,L\}} (\tau^n + \sum_{j \in \{r,s\}} \phi_1^{j,n}) - d^k + \sum_{n \in \{M,I,L\}} \phi_2^{k,n} & \text{if } k = r \\ \tilde{W}^0 - \sum_{n \in \{M,I,L\}} (\tau^n + \sum_{j \in \{r,s\}} \phi_1^{j,n}) - d^k + \sum_{n \in \{M,I,L\}} \phi_2^{k,n} & \text{if } k = s \end{cases}$$

$$\tilde{p}^k = \begin{cases} (1 - p^r)^{\eta^r \cdot l} \cdot (1 - p^s)^{\eta^s \cdot l} & \text{if } k = 0 \\ 1 - (1 - p^r)^{\eta^r \cdot l} & \text{if } k = r \\ (1 - p^r)^{\eta^r \cdot l} \cdot [1 - (1 - p^s)^{\eta^s \cdot l}] & \text{if } k = s \end{cases}$$

In the above, V represents the expected utility from car ownership; $\tilde{p}^k$ represents the probability of occurrence of state $k \in \{0, r, s\}$[4]; p^j represents the probability of a crash per mile driven on portion $j \in \{r, s\}$ of the roadway; $\tilde{W}^0$ is consumers' wealth; d^j represents cost of damages incurred as a result of the accident on portion j of the roadway; τ^n represents the technology-related cost paid to agent $n \in \{M, I, L\}$; $\phi_1^{j,n}$ represents the premium that consumers pay to agent n for coverage on portion j of the roadway; $\phi_2^{j,n}$ represents insurance payout from agent n to the victim as a result of an accident on portion j of the roadway; l is the length of the roadway; η^j represents the fraction of the roadway of type j; and $U(\cdot)$ describes utility as a function of wealth. Naturally, $\tau^L = 0$ since insurers do not provide any driving related technology; and $\phi_1^{r,I} = \phi_2^{r,I} = 0$ since ISSPs do not operate on the regular portion of the roadway. Figure 2 shows a schematic of these interactions.

Moreover, because d^j is constant and known in advance by all parties, all risks is contractible.

[4]The form of $\tilde{p}^k$ implies a sequence in the occurrence of accidents: travellers first travel on the regular portion of the road and, if uninjured, travel on the smart portion of the road. Other configurations are possible, though this one is most straightforward.

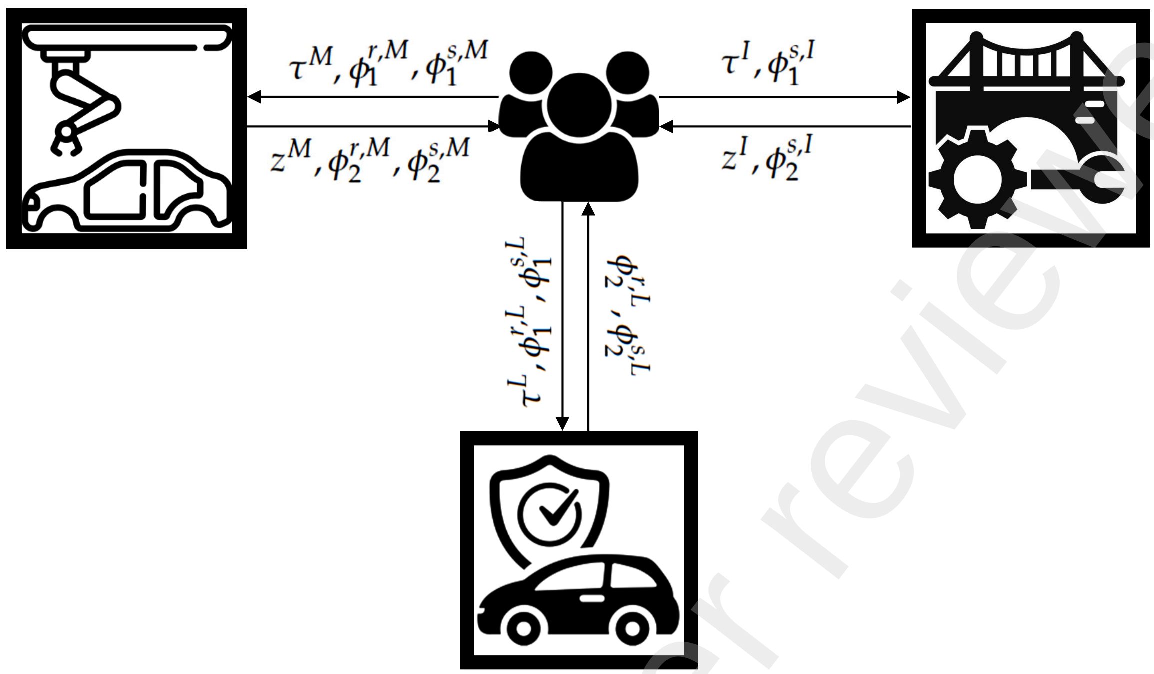

Figure 2: *Schematic of interactions between consumers and manufacturer, ISSP, and insurer.*

However, the insights we derive here carry over to situations in which accidents types and damages are heterogeneous, provided a contract can be drawn up for every possible occurrence.

We assume that customers are heterogeneous in their reservation values (but not in risk profile) for AV ownership which is distributed in the population according to a CDF $\Lambda(\cdot)$. This assumption gives rise to a demand function for AVs, $\lambda = \lambda^0 \cdot \Lambda(V)$, where λ^0 is the market size. We make the following assumptions on $U(\cdot)$:

Assumption 1. *We assume the following:*

A1.1 *Utility is strictly increasing in wealth:* $U' > 0$.

A1.2 *Customers experience decreasing absolute risk aversion (DARA):* $U'' < 0$ *and* $U''' > 0$.

The crash probabilities are influenced by the decisions of the manufacturer and the ISSP as follows:

$$p^r = P^0 \cdot P(z^M, 0)$$
$$p^s = P^0 \cdot P(z^M, z^I)$$

where z^M is the quality level of AV technology; z^I is the quality level on the smart portion of the roadway; P^0 is the crash probability when driving without automation and digitalization; and $P(\cdot,\cdot)$ is a probability function.

Assumption 2. *We assume the following:*

A2.1 *The crash probability is strictly decreasing in technology quality:* P_1, $P_2 < 0$.

A2.2 $P(0,0) = 1$.

The manufacturer and the ISSP face cost c^M per vehicle and c^I per mile of roadway equipped, respectively. These costs depend on technology levels as follows:

$$c^M = C^M(z^M)$$
$$c^I = C^I(z^I)$$

Assumption 3. *We assume the following:*

A3.1 *The cost functions are strictly increasing in technology quality: $C^{M'}, C^{I'} > 0$.*

A3.2 *The cost functions are strictly convex in technology quality: $C^{M''}, C^{I''} > 0$.*

We can then describe our problem using the system of equations below:

$$\lambda = \lambda^0 \cdot \Lambda(V) \tag{1a}$$

$$V = \tilde{p}^0 \cdot U(W^0) + \tilde{p}^r \cdot U(W^r) + \tilde{p}^s \cdot U(W^s) \tag{1b}$$

$$W^k = \begin{cases} \tilde{W}^0 - \sum_{n\in\{M,I,L\}}(\tau^n + \sum_{j\in\{r,s\}} \phi_1^{j,n}) & \text{if } k = 0 \\ \tilde{W}^0 - \sum_{n\in\{M,I,L\}}(\tau^n + \sum_{j\in\{r,s\}} \phi_1^{j,n}) - d^k + \sum_{n\in\{M,I,L\}} \phi_2^{k,n} & \text{if } k = r \\ \tilde{W}^0 - \sum_{n\in\{M,I,L\}}(\tau^n + \sum_{j\in\{r,s\}} \phi_1^{j,n}) - d^k + \sum_{n\in\{M,I,L\}} \phi_2^{k,n} & \text{if } k = s \end{cases} \tag{1c}$$

$$\tilde{p}^k = \begin{cases} (1-p^r)^{\eta^r \cdot l} \cdot (1-p^s)^{\eta^s \cdot l} & \text{if } k = 0 \\ 1 - (1-p^r)^{\eta^r \cdot l} & \text{if } k = r \\ (1-p^r)^{\eta^r \cdot l} \cdot [1 - (1-p^s)^{\eta^s \cdot l}] & \text{if } k = s \end{cases} \tag{1d}$$

$$p^r = P^0 \cdot P(z^M, 0) \tag{1e}$$

$$p^s = P^0 \cdot P(z^M, z^I) \tag{1f}$$

$$c^M = C^M(z^M) \tag{1g}$$

$$c^I = C^I(z^I) \tag{1h}$$

We now consider three different scenarios to understand:

1. the socially optimal apportionment of liability among the four agents in our model;
2. the effect of market structure on liability apportionment;
3. and whether the presence of the auto insurance industry improves outcomes.

3.2 First-best

We consider the social welfare maximization problem:

$$\begin{aligned} \max_{\substack{\tau^n, z^M, z^I \\ \phi_1^{j,n}, \phi_2^{j,n}}} \quad & \lambda^0 \cdot \int_0^V \Lambda(x) \cdot dx + \left[\left(\tau^M + \sum_{j\in\{r,s\}} (\phi_1^{j,M} - \tilde{p}^j \cdot \phi_2^{j,M}) - c^M \right) \cdot \lambda \right] + \\ & \left[\left(\tau^I + \phi_1^{s,I} - \tilde{p}^s \cdot \phi_2^{s,I} \right) \cdot \lambda - c^I \cdot \eta^s \cdot l \right] + \left[\left(\sum_{j\in\{r,s\}} \phi_1^{j,L} - \tilde{p}^j \cdot \phi_2^{j,L} \right) \cdot \lambda \right] \\ \text{s.t.} \quad & \text{Equations (1a) to (1h)} \end{aligned} \tag{SO}$$

In this problem, given the fraction of the roadway allocated to smart infrastructure, a social planner essentially minimizes the social cost of driving, taking into account the availability of insurance (either through automakers and ISSPs or through legacy insurers). We implicitly assume that insurers, automakers, and ISSPs are risk neutral so that the expected payouts from accidents enter the objective function linearly.

The first-order necessary conditions (FONCs) for optimality yield (more details on the derivation are given in Appendix A):

$$\hat{\phi}^j = d^j \quad \forall j \in \{r, s\} \tag{2a}$$

$$\hat{\tau} = c^M + \sum_{j \in \{r,s\}} \tilde{p}^j \cdot \hat{\phi}^j + \frac{\Lambda}{\Lambda'} \cdot \left[\frac{1}{U'(W^0)} - 1\right] \tag{2b}$$

$$-\frac{C^{M'}(z^M)}{P^0 \cdot l} = (d^r - d^s) \cdot \frac{P_1(z^M, 0) \cdot (1 - \tilde{p}^r) \cdot \eta^r}{1 - p^r} + \tilde{p}^0 \cdot d^s \cdot \left[\frac{P_1(z^M, z^I) \cdot \eta^s}{1 - p^s} + \frac{P_1(z^M, 0) \cdot \eta^r}{1 - p^r}\right] \tag{2c}$$

$$-\frac{C^{I'}(z^I)}{P^0 \cdot \lambda} = \tilde{p}^0 \cdot d^s \cdot \frac{P_2(z^M, z^I)}{1 - p^s} \tag{2d}$$

where:

- $\hat{\tau} = \sum_{n \in \{M,I,L\}} (\tau^n + \sum_{j \in \{r,s\}} \phi_1^{j,n})$ is the cost of vehicle ownership, including both purchase, road usage, and insurance premium costs;
- $\hat{\phi}^j = \sum_{n \in \{M,I,L\}} \phi_2^{j,n}$ is the total coverage that a vehicle owner receives in case of an accident on portion j of the roadway.

Firstly, we note that longer travel and higher demand will lead to increased investment in automation and digitalization technology, respectively (Equations (2c) and (2d)). Indeed, more travel increases the individual risk of accident and thus, in response, the planner invests more in vehicle technology. Additionally, increased travel volume increases the expected losses from an accident, thus inducing the planner to invest more in infrastructure technology. When it comes to the relationship between vehicle and infrastructure technology, we note that when an increase in infrastructure technology reinforces (weakens) the safety effect of automation, $P_{12} < 0$ ($P_{21} > 0$), then higher equilibrium digitalization implies higher (lower) spending c^M on vehicle technology. Thus, substitutability and complementarity matter in the optimal provision of digitalization and automation. Those insights are in keeping with the results from Vignon et al. (2022).

Second, from a societal perspective, full coverage regardless of accident type is optimal (Equation (2a)). Both on the smart and regular portions of the roadway, combined insurance payouts are sufficient to cover all damages to customers in the event of a crash. Equation (2b) indicates that the cost of vehicle ownership is adjusted to reflect producers' liability exposure and now includes the expected producer losses from vehicle operation. Thus, even when automakers decide to shoulder losses resulting from an AV accident, vehicle ownership cost might not decrease, except if both crash probability and/or crash severity significantly decrease. Moreover, while optimal digitalization provision solely depends on accident severity on the smart road (Equation (2d)), so that an increase in d^s increases the optimal z^I, optimal automation provision depends on severity on both road segments (Equation (2c)). Indeed, the marginal cost of automation technology is equalized with the sum of three terms:

- $d^r \cdot \frac{P_1(z^M,0)\cdot(1-\tilde{p}^r)\cdot\eta^r}{1-p^r}$ which captures the marginal reduction in expected payouts from accidents on the regular portion of the road due to a marginal increase in automation technology;
- $\tilde{p}^0 \cdot d^s \cdot \left[\frac{P_1(z^M,z^I)\cdot\eta^s}{1-p^s} + \frac{P_1(z^M,0)\cdot\eta^r}{1-p^r}\right]$ which captures the marginal reduction in expected payouts from accidents on the smart portion of the road due to a marginal increase in automation technology;
- and $-d^s \cdot \frac{P_1(z^M,0)\cdot(1-\tilde{p}^r)\cdot\eta^r}{1-p^r}$ which captures the fact that an increase in automation technology, by decreasing the probability of accidents on the regular portion of the road, increases the odds of incurring an accident on the smart portion of the road, thus raising the expected cost of accidents on the smart portion.

While the first two terms described above capture the direct effects of automation technology on liability exposure, the third term captures a second-order effect. This second-order effect is likely negligible at the first-best, so that an increase in either d^r or d^s would result in an increase in the optimal automation level.

Thirdly, from a social perspective, when all providers are risk-neutral, the sharing of liability between automakers, ISSPs, and insurers is irrelevant, so long as full coverage obtains. It is the total coverage available to customers which determines optimal safety levels at the first-best, not a particular allocation of liability among the parties. However, as we will later see in Section 3.3, the manner in which liability is apportioned will matter for safety in practice.

Lastly, at optimality, joint producer profit, π^*, is given by:

$$\pi^* = \frac{\Lambda}{\Lambda'} \cdot \left[\frac{1}{U'(W^0)} - 1\right] \cdot \lambda - c^I \cdot \eta^s \cdot l = \frac{\Lambda}{\Lambda'} \cdot \left(\left[-\frac{A(W^0)}{U''(W^0)}\right] - 1\right) \cdot \lambda - c^I \cdot \eta^s \cdot l \qquad (3)$$

where $A(\cdot) > 0$ is the absolute risk aversion function. From the above, it is clear that a key driver of profitability for producers is customers' risk aversion. When risk aversion is low, $\pi^* < 0$ and either the provision of infrastructure services requires subsidies or a regulated monopoly. However, when risk aversion is sufficiently high, the provision of automation and infrastructure services can be profitable. In essence, it is users' risk aversion towards accidents that helps bankroll the planner in the provision of smart infrastructure: users are willing to pay more to reduce the risk involved in driving. Thus, in addition to the optimal provision of capacity and optimal tolling on congested roads (Verhoef and Rouwendal, 2004), optimal insurance in the presence of strong risk aversion can also lead to self-financing for smart road infrastructure.

3.3 Unregulated environment

Here, we would like to understand how the provision of safety and insurance in the absence of regulations differs from the socially optimal allocation. However, we must account for the fact that this unregulated equilibrium will differ from the first-best due to different forces. First, producers could wield market power on their respective offerings (automation for automakers, digitalization for ISSPs, insurance for legacy insurers), thus distorting both safety and premiums from their optimal levels. Moreover, in the case in which they operate independently, lack of coordination could itself distort offerings, as demonstrated in Vignon et al. (2022). As a result, we must isolate each of these channels and understand their contributions to sub-optimality. Thus, to investigate the effect of market power, we begin by first analyzing an unregulated, integrated monopoly which provides automation, digitalization, and insurance. Then, to understand the effects of non-cooperation, we investigate the Nash game between automaker, ISSP, and insurers. We note that each of these configurations, in and of itself, could emerge in practice. Indeed, an integrated consortium formed of automakers, ISSPs, insurers, and government agencies and that manages both vehicle and infrastructure standards could easily emerge, judging from a few existing examples (e.g.: Grinell (2020)). However, forming such a consortium at a large-scale might not be readily possible, given the disparity in regulation and willingness to entertain smart mobility that prevails across states, whence the value of studying an uncooperative setting.

3.3.1 Integrated monopoly

We now consider what happens when a single entity manages the automaker, the ISSP, and insurance provision. This monopolist maximizes profits according to the following:

$$\begin{aligned} \max_{\hat{\tau},\, z^M,\, z^I,\, \hat{\phi}^j} \quad & (\hat{\tau} - c^M - \sum_{j\in\{r,s\}} \tilde{p}^j \cdot \hat{\phi}^j) \cdot \lambda - c^I \cdot \eta^s \cdot l \\ \text{s.t.} \quad & \text{Equations (1a) to (1h)} \end{aligned} \tag{MO}$$

where $\hat{\tau}$ and $\hat{\phi}^j$ are defined as in Equation (2). Assuming an interior solution exists, the FONC for (MO) yields:

$$\hat{\phi}^j = d^j \quad \forall j \in \{r,s\} \tag{4a}$$

$$\hat{\tau} = c^M + \sum_{j\in\{r,s\}} \tilde{p}^j \cdot \hat{\phi}^j + \frac{\Lambda}{\Lambda'} \cdot \Big[-\frac{A(W^0)}{U''(W^0)} \Big] \tag{4b}$$

$$-\frac{C^{M'}(z^M)}{P^0 \cdot l} = (d^r - d^s) \cdot \frac{P_1(z^M, 0)\cdot(1-\tilde{p}^r)\cdot\eta^r}{1-p^r} + \tilde{p}^0 \cdot d^s \cdot \Big[\frac{P_1(z^M, z^I)\cdot\eta^s}{1-p^s} + \frac{P_1(z^M,0)\cdot\eta^r}{1-p^r} \Big] \tag{4c}$$

$$-\frac{C^{I'}(z^I)}{P^0 \cdot \lambda} = \tilde{p}^0 \cdot d^s \cdot \frac{P_2(z^M, z^I)}{1-p^s} \tag{4d}$$

Here, the monopolist covers all potential damages, just as in the first-best (Equation (4a)). However, due to market power, this naturally occurs at a higher vehicle ownership cost than that chosen by the planner (Equation (4b)). This leads to lower demand and suboptimal safety levels (Equations (4c) and (4d)). Moreover, it is clear that full coverage could obtain even in the absence of legacy auto insurers.

3.3.2 Generalized Nash Equilibrium

What happens, however, when we allow for independent operation for all three entities? To answer this question, we consider the Nash game between all three entities. Letting $\mathbf{a}^n = \{\tau^n, \phi^{r,n}, \phi^{s,n}, z^n\}$ denote the action vector of player $k \in \{M, I, L\}$[5], $\{\mathbf{a}^{M,*}, \mathbf{a}^{I,*}, \mathbf{a}^{L,*}\}$ constitutes a Generalized Nash Equilibrium (GNE) if $\mathbf{a}^{M,*}$ solves the following problem, taking $\mathbf{a}^{I,*}$ and $\mathbf{a}^{L,*}$ as given:

[5]Recall that $\tau^L = 0$, $z^L = 0$, and $\phi_1^{r,I} = \phi_2^{r,I} = 0$

$$\begin{aligned}
&\max_{\hat{\tau}^M,\, z^M,\, \phi_2^{j,M}} \quad \Big(\hat{\tau}^M - c^M - \sum_{j\in\{r,s\}} \tilde{p}^j \cdot \phi_2^{j,M}\Big) \cdot \lambda \\
&\text{s.t.} \qquad \text{Equations (1a) to (1h)}
\end{aligned} \tag{N-M}$$

$\mathbf{a}^{I,*}$ solves the following problem, taking $\mathbf{a}^{M,*}$ and $\mathbf{a}^{U,*}$ as given:

$$\begin{aligned}
&\max_{\hat{\tau}^I,\, z^I,\, \phi_2^{j,I}} \quad (\hat{\tau}^I - \tilde{p}^s \cdot \phi_2^{s,I}) \cdot \lambda - c^I \cdot \eta^s \cdot l \\
&\text{s.t.} \qquad \text{Equations (1a) to (1h)}
\end{aligned} \tag{N-I}$$

and $\mathbf{a}^{L,*}$ solves the following problem, taking $\mathbf{a}^{M,*}$ and $\mathbf{a}^{I,*}$ as given:

$$\begin{aligned}
&\max_{\hat{\tau}^L,\, \phi_2^{j,L}} \quad \Big(\hat{\tau}^L - \sum_{j\in\{r,s\}} \tilde{p}^j \cdot \phi_2^{j,L}\Big) \cdot \lambda \\
&\text{s.t.} \qquad \text{Equations (1a) to (1h)}
\end{aligned} \tag{N-L}$$

where $\hat{\tau}^n = \tau^n + \sum_{j\in\{r,s\}} \phi_1^{j,n}$ for $n \in \{M, I, L\}$.

Using the FONCs of (N-M), (N-I), and (N-L), we derive the following equations characterizing the equilibrium:

$$\hat{\phi}^j = d^j \quad \forall j \in \{r,s\} \tag{5a}$$

$$\hat{\tau}^M = c^M + \sum_{j\in\{r,s\}} \tilde{p}^j \cdot \phi_2^{j,M} + \frac{\Lambda}{\Lambda'} \cdot \Big[-\frac{A(W^0)}{U''(W^0)}\Big] \tag{5b}$$

$$\hat{\tau}^I = \phi_2^{s,I} \cdot \tilde{p}^s + \frac{\Lambda}{\Lambda'} \cdot \Big[-\frac{A(W^0)}{U''(W^0)}\Big] \tag{5c}$$

$$\hat{\tau}^L = \sum_{j\in\{r,s\}} \tilde{p}^j \cdot \phi_2^{j,L} + \frac{\Lambda}{\Lambda'} \cdot \Big[-\frac{A(W^0)}{U''(W^0)}\Big] \tag{5d}$$

$$-\frac{C^{M'}(z^M)}{P^0 \cdot l} = (\phi_2^{r,M} - \phi_2^{s,M}) \cdot \frac{P_1(z^M,0)\cdot(1-\tilde{p}^r)\cdot\eta^r}{1-p^r} + \tilde{p}^0 \cdot \phi_2^{s,M} \cdot \Big[\frac{P_1(z^M,z^I)\cdot\eta^s}{1-p^s} + \frac{P_1(z^M,0)\cdot\eta^r}{1-p^r}\Big] \tag{5e}$$

$$-\frac{C^{I'}(z^I)}{P^0 \cdot \lambda} = \tilde{p}^0 \cdot \phi_2^{s,L} \cdot \frac{P_2(z^M,z^I)}{1-p^s} \tag{5f}$$

Here too, full coverage is provided to AV owners on both portions of the road (Equation (5a)). However, this occurs at a higher total cost than under the first-best and the integrated monopoly because of triple marginalization (Equations (5b) to (5d)). Moreover, which entity provides that coverage is unclear. As such, multiple equilibria might exist, ranging from one or two of the entities providing little coverage to all entities sharing the liability equally. Some of these equilibria would be problematic from a safety viewpoint. Indeed, consider the following rewriting of Equations (5e) and (5f):

$$-C^{M'}(z^M) = (d^r - \phi_2^{r,L}) \cdot \frac{\partial \tilde{p}^r}{\partial z^M}(z^M,0) + [d^s - (\phi_2^{s,L} + \phi_2^{s,I})] \cdot \frac{\partial \tilde{p}^s}{\partial z^M}(z^M,z^I)$$

$$-\frac{C^{I'}(z^I)}{P^0 \cdot \lambda} = \tilde{p}^0 \cdot \phi_2^{s,L} \cdot \frac{P_2(z^M, z^I)}{1 - p^s}$$

We have effectively eliminated $\phi_2^{j,M}$ from Equations (5e) and (5f) using the fact that full coverage obtains. It is straightforward to see that allocations in which $\phi_2^{j,L} > 0$ or $\phi_2^{s,I} > 0$ would automatically result in lower automation levels than the first-best. This is because, for the automaker, any assumption of liability from either the insurer or the automaker reduces the incentive for safety investment in automation. This mainly occurs because of lack of coordination between all three insurance providers. Thus, even though full coverage obtains, the total expected cost of accidents would simply be higher than socially optimal.

In addition to this first-order distortion effect, another distortion effect that might arise is related to complementarity between automation and digitalization. If infrastructure technology acts as a substitute for vehicle technology, then an equilibrium with low $\phi^{s,M}$ would result in lower (higher) equilibrium automation (digitalization) technology than an equilibrium with high $\phi^{s,M}$. In the case of complementarity between the two technologies, lower $\phi^{s,M}$ could induce both lower automation and digitalization at equilibrium. In the absence of any guiding principle, either of these equilibria might prevail.

Thus, the planner might need to establish a regulation, in the manner of Di et al. (2020), to ensure that no undesirable equilibrium is reached (e.g.: one of the agents free-riding from the others' quality investment or on insurers' risk assumption). We discuss the design of these rules in the following section.

3.4 Liability rule implementation

Here, we investigate whether a liability rule can prevent the Nash game from settling at an undesirable equilibrium. First, we note that an appropriate liability rule in this context will be negligence-based (Talley, 2019; Di et al., 2020), thus making the liability share of the automaker and the ISSP dependent on their respective level of safety investment. However, when negligence-based liability is considered, how much insurers should shoulder becomes unclear since they do not make any safety decisions. A straightforward assumption, then, and one that has often been used in the literature, is that automakers (and ISSPs, when present) jointly shoulder all the liability.

In this section, we first consider this special case before investigating a regime in which the total burden that falls on automaker and ISSPs is determined by their joint investment levels and insurers step in to cover the rest.

3.4.1 Legacy insurers do not provide any coverage

We introduce $\alpha = \mathcal{A}(z^M, z^I)$, the share function for the manufacturer. α determines the fraction of total damages that the manufacturer must shoulder when an accident occurs on the smart portion of the road. Letting $z^{M,**}$ and $z^{I,**}$ denote first-best technology levels, $\{\mathbf{a}^{M,*}, \mathbf{a}^{I,*}\}$ constitutes a Generalized Nash Equilibrium (GNE) if $\mathbf{a}^{M,*}$ solves the following problem, taking $\mathbf{a}^{I,*}$ as given:

$$
\begin{aligned}
\max_{\hat{\tau}^M,\, z^M,\, \phi_2^{j,M}} \quad & (\hat{\tau}^M - c^M - \sum_{j\in\{r,s\}} \tilde{p}^j \cdot \phi_2^{j,M}) \cdot \lambda \\
\text{s.t.} \quad & \text{Equations (1a) to (1h),} \\
& \phi_2^{s,M} \cdot \lambda \geq \mathcal{A}\Big(z^{M,**} - z^M, z^{I,**} - z^I\Big) \cdot d^s \cdot \lambda
\end{aligned}
\tag{N-MR}
$$

and $\mathbf{a}^{I,*}$ solves the following problem, taking $\mathbf{a}^{M,*}$ as given:

$$
\begin{aligned}
\max_{\hat{\tau}^I,\, z^I,\, \phi_2^{s,I}} \quad & (\hat{\tau}^I - \tilde{p}^s \cdot \phi^{s,I}) \cdot \lambda - c^I \cdot \eta^s \cdot l \\
\text{s.t.} \quad & \text{Equations (1a) to (1h),} \\
& \phi_2^{s,I} \cdot \lambda \geq \Big[1 - \mathcal{A}\Big(z^{M,**} - z^M, z^{I,**} - z^I\Big)\Big] \cdot d^s \cdot \lambda
\end{aligned}
\tag{N-IR}
$$

(In the above, pre-multiplying by λ simplifies a lot of the subsequent calculus).

Assumption 4. *We make the following assumptions on $\mathcal{A}(\cdot,\cdot)$:*

A4.1 *The share function is differentiable.*

A4.2 *The share function is strictly decreasing in the automaker's choice of automation: $\mathcal{A}_1 > 0$*[6].

A4.3 *The share function is strictly increasing in the ISSP's choice of digitalization: $\mathcal{A}_2 < 0$.*

A4.4 *The share function is strictly positive and below 1: $0 < \mathcal{A}(\cdot,\cdot) < 1$.*

It is important to note that the shape and functional form of $\mathcal{A}(\cdot,\cdot)$ will differ depending on the equilibrium targeted by the planner. In some instances, for example, the desired target might yield $\mathcal{A}(0,0) = \frac{1}{2}$, so that equal sharing in liability is the effective desired target. In other instances, however, this might change. Regardless, by our assumptions, both agents will always hold some liability, however infinitesimal.

By deriving the FONC for N-MR and N-IR, we can first show that the regime with a liability rule results in greater safety levels than the one without. Additionally, by assuming Equation (2c) and Equation (2d) hold, we can derive sufficient conditions for a share function to replicate the desired quality levels [7]. Indeed, let $\mu^k \geq 0$ denote the Lagrangian multiplier associated with the share constraint for player k. Then, sufficient conditions for the GNE to yield $\{z^{M,**}, z^{I,**}\}$ are that there exists $\mu^M > 0$ and $\mu^I > 0$ satisfying the following two equations:

$$
[1 - \mathcal{A}(0,0)] \cdot \frac{d\tilde{p}^s}{dz^M}(z^{M,**}, z^{I,**}) + \mu^M \cdot \mathcal{A}_1(0,0) = 0
$$

$$
\Big([1 - \mathcal{A}(0,0)] \cdot \lambda^* - \lambda^{**}\Big) \cdot \frac{d\tilde{p}^s}{dz^I}(z^{M,**}, z^{I,**}) + \mu^I \cdot \mathcal{A}_2(0,0) \cdot \lambda^* = 0
$$

where

$$
\frac{1}{P^0 \cdot l} \cdot \frac{d\tilde{p}^s}{dz^M}(z^{M,**}, z^{I,**}) = -\frac{P_1(z^{M,**}, 0) \cdot (1 - \tilde{p}^r) \cdot \eta^r}{1 - p^r} + \tilde{p}^0 \cdot \Big[\frac{P_1(z^{M,**}, z^{I,**}) \cdot \eta^s}{1 - p^s} + \frac{P_1(z^{M,**}, 0) \cdot \eta^r}{1 - p^r}\Big]
$$

[6] Note that $\mathcal{A}(\cdot,\cdot)$ increases with $-z^M$ and $-z^I$

[7] The derivation is more clearly spelled out in Appendix A

$$\frac{1}{P^0 \cdot \eta^s \cdot l} \cdot \frac{d\tilde{p}^s}{dz^I}(z^{M,**}, z^{I,**}) = \tilde{p}^0 \cdot \frac{P_2(z^{M,**}, z^{I,**})}{1-p^s}$$
$$\lambda^{**} > \lambda^*$$

and λ^* is demand under Nash. By Assumptions **A4.2**, **A4.3** and **A4.4**, and assuming that the first-order effects of automation technology dominate the second-order effects, the sufficiency condition is always met. Thus, an appropriate liability rule can reproduce the first-best automation and digitalization levels (though at a higher price than the first-best). In practice, however, implementing such a rule would be costly. Indeed, it would involve expanding resources to determine the quality levels of technology used by both entities, *a posteriori*. In the presence of certain standards, this cost could be reduced and quality could be ensured *ex ante*.

3.4.2 Legacy insurers provide coverage

Here, we now introduce $\beta^j = \mathcal{B}^j(z^M, z^I)$, the total share function for accidents on the j-th portion of the road. It is the fraction of total damages on said portion that automakers and ISSP must jointly cover.

Now, $\{\mathbf{a}^{M,*}, \mathbf{a}^{I,*}, \mathbf{a}^{L,*}\}$ constitutes a Generalized Nash Equilibrium (GNE) if $\mathbf{a}^{M,*}$ solves the following problem, taking $\{\mathbf{a}^{I,*}, \mathbf{a}^{L,*}\}$ as given:

$$\begin{aligned} \max_{\hat{\tau}^M,\, z^M, \phi_2^{j,M}} \quad & (\hat{\tau}^M - c^M - \sum_{j\in\{r,s\}} \tilde{p}^j \cdot \phi_2^{j,M}) \cdot \lambda \\ \text{s.t.} \quad & \text{Equations (1a) to (1h),} \\ & \phi^{j,M} \cdot \lambda \geq \mathcal{A}\Big(z^{M,**} - z^M, z^{I,**} - z^I\Big) \cdot \mathcal{B}^j\Big(z^{M,**} - z^M, z^{I,**} - z^I\Big) \cdot d^j \cdot \lambda \quad \forall j \in \{r,s\} \end{aligned} \tag{N-MR2}$$

, $\mathbf{a}^{I,*}$ solves the following problem, taking $\{\mathbf{a}^{M,*}, \mathbf{a}^{L,*}\}$ as given:

$$\begin{aligned} \max_{\hat{\tau}^I,\, z^I, \phi_2^{s,I}} \quad & (\hat{\tau}^I - \tilde{p}^s \cdot \phi_2^{s,I}) \cdot \lambda - c^I \cdot \eta^s \cdot l \\ \text{s.t.} \quad & \text{Equations (1a) to (1h),} \\ & \phi_2^{s,I} \cdot \lambda \geq \Big[1 - \mathcal{A}\Big(z^{M,**} - z^M, z^{I,**} - z^I\Big)\Big] \cdot \mathcal{B}^s\Big(z^{M,**} - z^M, z^{I,**} - z^I\Big) \cdot d^s \cdot \lambda \end{aligned} \tag{N-IR2}$$

and $\mathbf{a}^{L,*}$ solves the following problem, taking $\{\mathbf{a}^{M,*}, \mathbf{a}^{I,*}\}$ as given:

$$\begin{aligned} \max_{\hat{\tau}^L,\, \phi_2^{j,L}} \quad & \Big(\hat{\tau}^L - \sum_{j\in\{r,s\}} \tilde{p}^j \cdot \phi_2^{j,L}\Big) \cdot \lambda \\ \text{s.t.} \quad & \text{Equations (1a) to (1h),} \\ & \phi_2^{j,L} \cdot \lambda \geq \Big[1 - \mathcal{B}^j\Big(z^{M,**} - z^M, z^{I,**} - z^I\Big)\Big] \cdot d^j \cdot \lambda \quad \forall j \in \{r,s\} \end{aligned} \tag{N-LR2}$$

Assumption 5. *We assume the following on $\mathcal{B}^j(\cdot,\cdot)$:*

A5.1 *The total share function is differentiable.*

A5.2 *The total share function is strictly decreasing in the automaker's and ISSP's choices of automation:* $\mathcal{B}^j_1 > 0$, $\mathcal{B}^j_2 > 0$.

A5.3 *The total share function exhibits complementarity between automation and digitalization:* $\mathcal{B}^j_{21} > 0$, $\mathcal{B}^j_{12} > 0$.

A5.4 *If* $z^{n,**} \leq z^n \ \forall n \in \{M, I\}$, *then* $\mathcal{B}^j(\cdot,\cdot) < 1$.

Assumptions **A5.2**, **A5.3** and **A5.4** imply that, by proper coordination to increase safety, automaker and ISSP can reduce their total cost. Then, regular insurers pick up the slack, providing coverage for $(1-\beta^j)\cdot d^j$ in damages.

Here again, the equilibrium results in greater safety levels than the unregulated Nash game. Additionally, it is possible to derive sufficient conditions for the rule to result in the desired safety levels. Namely, there exist $\mu^{j,M} > 0$ and $\mu^I > 0$ satisfying the following equations:

$$\sum_j [1 - \mathcal{A}^j(0,0)\cdot\mathcal{B}^j(0,0)] \cdot \frac{d\tilde{p}^j}{dz^M}(z^{M,**}, z^{I,**}) + \mu^{j,M}\cdot\gamma^{j,M}(0,0) = 0$$

$$\left([1 - \mathcal{A}(0,0)\cdot\mathcal{B}^s(0,0)]\cdot\lambda^* - \lambda^{**}\right)\cdot\frac{d\tilde{p}^s}{dz^I}(z^{M,**}, z^{I,**}) + \mu^I\cdot\gamma^I(0,0) = 0$$

where:

$$\begin{aligned}
\gamma^{j,M}(\cdot,\cdot) &= \mathcal{A}^j_1(\cdot,\cdot)\cdot\mathcal{B}^j(\cdot,\cdot) + \mathcal{A}^j(\cdot,\cdot)\cdot\mathcal{B}^j_2(\cdot,\cdot)\\
\gamma^I(\cdot,\cdot) &= \mathcal{A}_2(\cdot,\cdot)\cdot\mathcal{B}^s(\cdot,\cdot) + \mathcal{A}(\cdot,\cdot)\cdot\mathcal{B}^s_2(\cdot,\cdot)\\
\mathcal{A}^r(\cdot,\cdot) &= 1\\
\mathcal{A}^s(\cdot,\cdot) &= \mathcal{A}(\cdot,\cdot)
\end{aligned}$$

This is always true provided we add, to Assumptions **A5.2**, **A5.3** and **A5.4**, an additional assumption: $\mathcal{A}_2(0,0)\cdot\mathcal{B}^s(0,0) + \mathcal{A}(0,0)\cdot\mathcal{B}^s_2(0,0) < 0$. That is, the effective share of damages borne by the automaker must be decreasing in the level of digitalization z^I.

3.5 Numerical example

This section provides more insights from numerical experiments. We first show scenarios in which there is an undesirable equilibrium that emerges in the absence of collaboration between the manufacturer, ISSP, and insurer. Then, we demonstrate how implementing regulations could achieve a desirable equilibrium. In this section, we assume that the AV demand function is:

$$\lambda = \frac{\lambda_0}{1 + 100\exp(-0.001V)}$$

, the utility function is:

$$U(W^k) = 5000\left(1 - \frac{1}{\exp(\frac{W^k}{1000})}\right)$$

Notation	Interpretation	Value
d^r	Cost of damages due to the accident on regular portion of the roadway (\$)	8000
d^s	Cost of damages due to the accident on smart portion of the roadway (\$)	5000
l	Roadway length / Mileage per trip (mi)	40
η^s	Smart portion of the roadway	0.2
λ^0	Base travel demand	5000
P^0	Base crash probability $(/mi)$	10^{-3}
W^0	Initial budget (\$)	10000

Table 2: *Parameter values for numerical examples*

and the probability function is:

$$P(z^M, z^I) = \exp(-0.25z^M - 0.1z^I)$$

and cost functions of manufacturer and ISSP are $c^M = 5(z^M)^2$ and $c^I = 20(z^I)^2$. Table 2 further presents the default value of the parameters used in this section.

Table 3 presents the optimal values of decision variables for the first-best and monopoly scenarios. As outlined in Sections 3.2 to 3.4, the monopoly scenario induces a slightly higher vehicle ownership cost due to market power, which results in lower demand in comparison to the first-best. This circumstance leads to suboptimal infrastructure automation quality. However, in both scenarios, there is full coverage of all damages in both regular and smart portions of the roadway.

Then, we consider a scenario in which three entities operate independently and their decisions result in a generalized Nash equilibrium. As discussed earlier, the equilibrium is not unique and there might exist multiple equilibria. Table 4 presents examples of such equilibria. In these cases, again there is full accident coverage on both regular and smart portions of the roadway. However, the proportion of coverage from each entity is different. For instance, in the equilibrium presented in the second row, all entities contribute to the coverage in case of a crash. However,the manufacturer does not provide any coverage in the equilibrium presented in the last row. We can also observe suboptimal safety levels as presented in the equilibria in the first, fifth, and sixth rows while the total vehicle ownership cost is higher compared to other equilibria (\$7996 vs. \$7185). Such occurrences show the importance of liability rules in preventing such undesirable equilibria.

Assuming that the insurers do not provide any coverage and taking the share function for the manufacturer as

$$\alpha = \mathcal{A}\left(z^{M,**} - z^M, z^{I,**} - z^I\right) = \frac{2}{2 + \exp(z^{M,**} - z^M) + \exp(-z^{I,**} + z^I)},$$

would lead to optimal safety levels and equal liability sharing in the smart portion of the roadway

	$\hat{\tau}$	z^M	z^I	$\hat{\phi}^r$	$\hat{\phi}^s$	λ	V
FB	6970.3	3.11	7.31	8000.0	5000.0	2691	4758.4
MO	7185	3.11	7.20	8000.0	5000.0	2619	4700.5

Table 3: *Optimal value of decision variables for the first-best and monopoly scenarios.*

	$\hat{\tau}^M$	z^M	$\phi_2^{r,M}$	$\phi_2^{s,M}$	$\hat{\tau}^I$	z^I	$\phi_2^{s,I}$	$\hat{\tau}^L$	$\phi_2^{r,L}$	$\phi_2^{s,L}$	λ	V
1	2772.0	2.99	8000.0	0	2607.0	6.53	0	2616.7	0	5000.0	2154	4326.2
2	2361.5	3.11	4001.9	1666.3	2410.9	7.20	1663.9	2412.6	3998.1	1669.8	2619	4700.5
3	2403.0	3.11	8000.0	213.9	2397.1	7.20	2365.2	2384.9	0	2420.8	2619	4700.5
4	1946.1	3.11	0	0	2645.1	7.20	3285.2	2593.8	8000.0	1714.8	2619	4700.5
5	2723.1	2.28	5098.6	255.5	2614.6	6.40	4070.1	2658.7	2901.4	674.3	2153	4325.8
6	2780.8	3.14	8000.0	3395.7	2607.7	1.64	802.2	2607.7	0	802.2	2153	4325.8

Table 4: *Optimal value of decision variables for different generalized Nash equilibria.*

between the manufacturer and ISSP as presented in the first row of Table 5. As can be seen, the liability rule results in the first-best automation and digitalization level, however, the total vehicle ownership cost is \$ 7185, which is higher than the first-best cost of \$ 6970.3. Now, we consider the case in which insurers provide coverage and assume that the manufacturer and ISSP's joint share function is:

$$\beta^j = \mathcal{B}^j\left(z^{M,**} - z^M, z^{I,**} - z^I\right) = \frac{1}{1+\exp(z^{M,**} - z^M)} \cdot \frac{1}{1+\exp(z^{I,**} - z^I)},$$

The resulting equilibrium is presented on the second row of Table 5. Again, the safety levels are equal to the first-best levels and higher than unregulated Nash. Moreover, all entities contribute to coverage in case of a crash due to the existence of the liability rules.

4 Uncertainty in accident costs

So far, we have provided answers to our first two guiding questions. To our first question regarding the effect of market structure on vehicle and infrastructure safety, we have shown that a setting in which automakers, ISSPs, and legacy insurers operate independently automatically leads to sub-optimal safety levels. Such an issue does not arise when a single entity produces vehicles, equips infrastructure, and provides insurance to consumers. Then, with respect to our second question, we have shown that, in the setting in which entities do not cooperate, it is possible to implement a liability rule that, under appropriate conditions, is sufficient to achieve first-best safety levels.

We have also shown that such a rule can be implemented whether or not legacy insurers remain in the market. This seems to imply that, under our assumptions, legacy auto insurers need not subsist in the age of automated driving. Since the market can be brought close to the social optimum with or without legacy insurers, it seems that they bring no advantage to consumers and have no means to differentiate themselves, at least in our setting. As such, they simply might cease to exist when automated driving becomes widespread.

$\hat{\tau}^M$	z^M	$\phi_2^{r,M}$	$\phi_2^{s,M}$	$\hat{\tau}^I$	z^I	$\phi_2^{s,I}$	$\hat{\tau}^L$	$\phi_2^{r,L}$	$\phi_2^{s,L}$	λ	V
3837.6	3.11	8000.0	2500.0	3347.4	7.31	2500.0	-	-	-	2619	4700.5
3303.0	3.11	1999.9	625.0	1023.2	7.31	625.0	2858.8	6000.1	3750.0	2619	4700.5

Table 5: *Optimal value of decision variables in the presence of liability rules.*

Now, however, we ask whether the presence of legacy auto insurers can improve outcomes (in terms of consumer welfare) in markets in which the severity of accidents, d^j, is not known ahead of time but is rather a random variable known with precision only after an accident has occurred. In this context, then, automakers and ISSPs might issue contracts with upper bounds on the amount recoverable for customers. Such a setting might also leave the door open for legacy insurers.

4.1 Model

Now, we consider a setting in which, even though only one type of accident may occur on any given portion of the roadway[8], there is uncertainty about the actual damages resulting from the accident. If we assume that automakers, ISSPs, insurers, and customers are willing and able to draw up a contract for every possible value of these damages, then the problem can most readily be recast as the one described in Appendix B. However, when this is not necessarily possible, as is the case in reality, automakers, ISSPs, and insurers might simply specify a maximum coverage that they are willing to provide. In this context, then, might the presence of legacy insurers improve outcomes?

For simplicity of exposition, we focus solely on the regular portion of the road. Similar mechanisms to those illustrated here will be at play on the smart portion of the road. Then the equations describing our problem are as follows:

$$\lambda = \lambda^0 \cdot \Lambda(V) \tag{11a}$$

$$\begin{aligned} V &= \tilde{p}^0 \cdot U(W^0) + \tilde{p}^r \cdot \left[G\left(\phi_2^{r,M} + \phi_2^{r,L}\right) \cdot U(W^0) + \int_{\phi_2^{r,M}+\phi_2^{r,L}}^{\infty} g(d^r) \cdot U\left(W^0 + \sum_{n\in\{M,L\}} \phi_2^{r,n} - d^r\right) \cdot dd^r \right] \\ &= \tilde{p}^0 \cdot U(W^0) + \tilde{p}^r \cdot G\left(\phi_2^{r,M} + \phi_2^{r,L}\right) \cdot U(W^0) + \\ &\quad \tilde{p}^r \cdot \left[\mathbb{E}\left[U\left(W^0 + \sum_{n\in\{M,L\}} \phi_2^{r,n} - d^r\right) \middle| d^r > \phi_2^{r,M} + \phi_2^{r,L} \right] \cdot \left(1 - G\left(\phi_2^{r,M} + \phi_2^{r,L}\right)\right) \right] \end{aligned} \tag{11b}$$

$$W^0 = \tilde{W}^0 - \sum_{n\in\{M,L\}} \hat{\tau}^n \tag{11c}$$

In the above, d^r, the cost of accidents on the regular portion of the road, is distributed according to a CDF G with corresponding PDF g. Essentially, the automaker provides coverage for accidents up to $\phi_2^{r,M}$. Then, the insurer steps in and provides additional coverage $\phi_2^{r,L}$. For accidents whose costs exceeds $\phi_2^{r,L} + \phi_2^{r,M}$, customers must cover the remaining costs. The profits for manufacturer and insurers can be written as:

$$\begin{aligned} \pi^M &= \left[\hat{\tau}^M - c^M - \tilde{p}^r \cdot \left(\int_0^{\phi_2^{r,M}} d^r \cdot g(d^r) \cdot dd^r + \phi_2^{r,M} \cdot [1 - G(\phi_2^{r,M})] \right) \right] \cdot \lambda \\ &= \left[\hat{\tau}^M - c^M - \tilde{p}^r \cdot \left(\mathbb{E}\left[d^r \middle| d^r \le \phi_2^{r,M} \right] \cdot G(\phi_2^{r,M}) + \phi_2^{r,M} \cdot [1 - G(\phi_2^{r,M})] \right) \right] \cdot \lambda \end{aligned}$$

[8] As in our original setting.

$$
\begin{aligned}
\pi^L &= \left[\hat{\tau}^L - \tilde{p}^r \cdot \Big(\int_{\phi_2^{r,M}}^{\sum_{n\in\{M,L\}} \phi_2^{r,n}} d^r \cdot g(d^r) \cdot dd^r + \phi_2^{r,L} \cdot \Big[1 - G(\sum_{n\in\{M,L\}} \phi_2^{r,n})\Big]\Big)\right] \cdot \lambda \\
&= \left[\hat{\tau}^L - \tilde{p}^r \cdot \mathbb{E}\Big[d^r \Big| \phi_2^{r,M} < d^r \leq \sum_{n\in\{M,L\}} \phi_2^{r,n}\Big] \cdot \Big(G(\sum_{n\in\{M,L\}} \phi_2^{r,n}) - G(\phi_2^{r,M})\Big)\right] \cdot \lambda + \\
&\quad \tilde{p}^r \cdot \phi_2^{r,L} \cdot [1 - G(\sum_{n\in\{M,L\}} \phi_2^{r,n})] \cdot \lambda
\end{aligned}
$$

4.2 Nash game

Solving for the equilibrium of the Nash game, we obtain the following:

$$
U'(W^0) = \frac{1 - G(\phi_2^{r,L} + \phi_2^{r,M})}{1 - G(\phi_2^{r,M})} \cdot \mathbb{E}\Big[U'(W^0 + \sum_{n\in\{M,L\}} \phi_2^{r,n} - d^r) | d^r > \sum_{n\in\{M,L\}} \phi_2^{r,n}\Big] \tag{13a}
$$

$$
\hat{\tau}^M = c^M + \tilde{p}^r \cdot \Big(\mathbb{E}[d^r | d^r \leq \phi_2^{r,M}] \cdot G(\phi_2^{r,M}) + \phi_2^{r,M} \cdot [1 - G(\phi_2^{r,M})]\Big) + \frac{\Lambda}{\Lambda'} \cdot \frac{1}{U'(W^0)} \tag{13b}
$$

$$
\begin{aligned}
\hat{\tau}^L = \tilde{p}^r \cdot \Big(&\mathbb{E}\Big[d^r \Big| \phi_2^{r,M} < d^r \leq \sum_{n\in\{M,L\}} \phi_2^{r,n}\Big] \cdot (G(\sum_{n\in\{M,L\}} \phi_2^{r,n}) - G(\phi_2^{r,M})) \\
&+ \phi_2^{r,L} \cdot [1 - G(\sum_{n\in\{M,L\}} \phi_2^{r,n})]\Big) + \frac{\Lambda}{\Lambda'} \cdot \frac{1}{U'(W^0)}
\end{aligned} \tag{13c}
$$

$$
-C^{M'}(z^M) = \frac{\partial \tilde{p}^r}{\partial z^M}(z^M, 0) \cdot \Big(\mathbb{E}[d^r | d^r \leq \phi_2^{r,M}] \cdot G(\phi_2^{r,M}) + \phi_2^{r,M} \cdot (1 - G(\phi_2^{r,M})\Big) \tag{13d}
$$

First, we note that the automation levels in this setting would differ from those under the socially optimal setting: the automaker does not consider the full expected cost of accidents in setting automation levels (Equation (13d)). Equation (13a) indicates that customers equalize the expected marginal utility of states with full coverage to that of states without full coverage. Additionally, Equation (13a) reveals that moving from a setting with $\phi_2^{r,L} \approx 0$ (no legacy insurer) to one with $\phi_2^{r,L} > 0$ may have two different effects depending on the shape of the marginal utility curve and of $G(\cdot)$. This in turn has ambiguous implications on the effect of legacy insurance in the market. Indeed, without legacy insurance, Equation (13a) becomes:

$$
U'(W^0) = \mathbb{E}\Big[U'(W^0 + \phi_2^{r,M} - d^r) | d^r > \phi_2^{r,M}\Big] \tag{14}
$$

The entry of the legacy insurer introduces the ratio $\frac{1-G(\phi_2^{r,L}+\phi_2^{r,M})}{1-G(\phi_2^{r,M})} < 1$ which puts a downward pressure on equilibrium W^0. However, holding $\phi_2^{r,M}$ constant, the expected marginal utility of uncovered states, $\mathbb{E}\Big[U'(W^0 + \sum_{n\in\{M,L\}} \phi_2^{r,n} - d^r) | d^r > \sum_{n\in\{M,L\}} \phi_2^{r,n}\Big]$, may increase or decrease. In the case of an increase, then there is an upward pressure on equilibrium wealth W^0. Now two questions remain in order to determine the effect of legacy insurance on the market. First, is the increase in marginal utility sufficient to increase in W^0, holding $\phi_2^{r,M}$ constant? Second, how does the automaker adjust $\phi_2^{r,M}$ in response to legacy entry?

Neither of these questions can be definitively answered *a priori* but is rather an empirical matter. Thus, it appears that one cannot rule out a positive impact from legacy insurance on the market, however infinitesimal. If such an impact were to materialize, it would portend that the industry need not disappear in the age of automated driving.

4.3 Numerical example

Here, $\eta^s = 0$, and the cost of accidents on the regular portion of the road is following a distribution, i.e., $d^r \sim Log\mathcal{N}(\mu, \sigma^2)$. To highlight the impact of uncertainty in accident costs, we solve for the equilibrium of the Nash game between the manufacturer and insurer when the accident cost is certain, ($d^r = 8000$), and compare it with the case when it is uncertain and distributed according to ($d^r \sim Log\mathcal{N}(8.9822, 0.01)$ or $d^r \sim Exp(0.000125)$). Figure 3 shows the PDF and CDF of accident costs, and Table 6 presents the Nash equilibrium results. We can see that automation levels are equal in cases with and without uncertainty in accident costs, and there exist multiple equilibria.

We now investigate the effect of legacy insurers on the market. When legacy insurers are absent, the manufacturer provides significant coverage (e.g.: \$15796 in the third row, resulting in over 99% of accidents being covered). When the insurer enters, the customers still receive significant coverage (e.g.: a total of \$13983 in the fourth row, resulting in over 99% of accidents being covered). While the manufacturer ends up shouldering a lesser burden, this does not impact safety levels. However, comparing rows 5 and 6, we observe that the introduction of the insurer significantly decreases utility and, as a consequence, total demand. This is likely due to significantly higher coverage than the expected value of accidents past $\phi_2^{r,M} = \$1264.4$ (the coverage provided by the manufacturer). This seems to imply that, under certain circumstances, the introduction of legacy insurers in the automated mobility market might actually harm consumers.

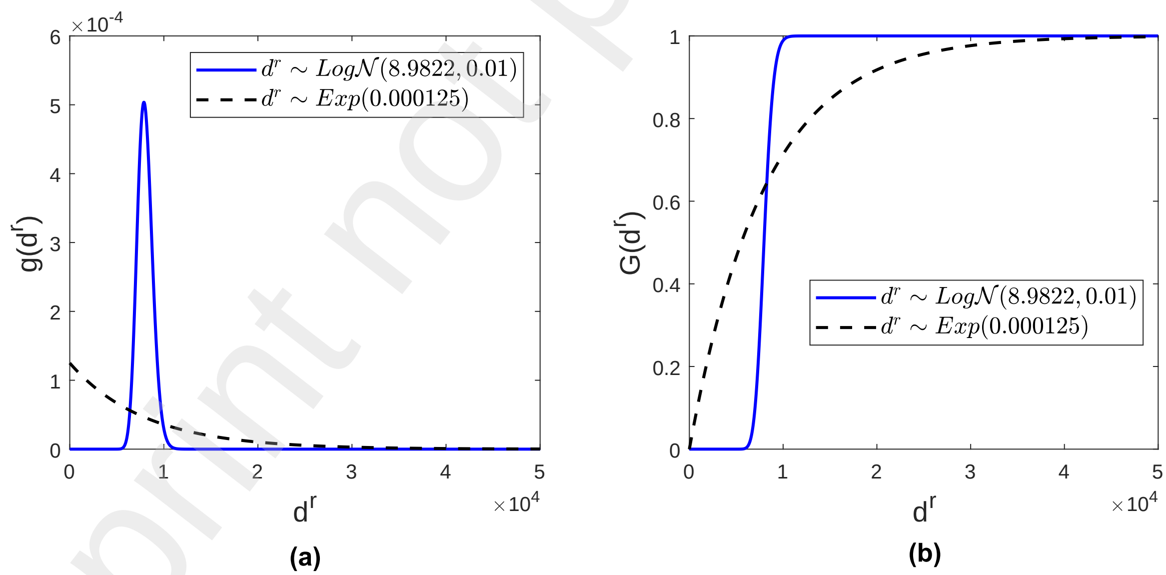

Figure 3: *(a) PDF and (b) CDF of uncertain accident costs distributions.*

	Accident cost	$\hat{\tau}^M$	z^M	$\phi_2^{r,M}$	$\hat{\tau}^L$	$\phi_2^{r,L}$	λ	V
1	$d^r = 8000$	7187.1	3.38	8000	-	-	2618	4699.8
2		3618.5	3.38	3380.5	3568.7	4619.5	2618	4699.8
3	$d^r \sim Log\mathcal{N}(8.9822, 0.01)$	7187.1	3.38	15796	-	-	2618	4699.8
4		3662.3	3.38	6362	3524.8	7621	2618	4699.8
5	$d^r \sim Exp(0.000125)$	7187.1	3.38	45936	-	-	2618	4699.8
6		3832.7	3.38	1264.4	3871.5	171948	2355.3	4489.3

Table 6: *Optimal value of decision variables for Nash equilibrium with and without uncertainty in accident cost.*

5 Conclusion

In this work, we investigated safety, liability, and insurance in an automated mobility market. We first showed that, in an unregulated setting, non-cooperation and competition between an automated vehicles manufacturer, an infrastructure service provider (ISSP), and a legacy insurer would lead to sub-optimal safety and insurance levels. Subsequently, we demonstrated that an appropriate liability rule could, if implemented, correct the discrepancy between first-best safety levels and safety levels in the unregulated market. Importantly, such a rule can be effective regardless of whether legacy insurers are present in the market. This raises the question as to whether legacy insurance has a role to play in an automated mobility market. Thus, our subsequent analysis showed that, in the case in which accident costs are uncertain, we cannot rule out that the presence of legacy insurers could benefit customers. However, numerical examples seem to indicate that the benefits, if any, could be marginal if not non-existent. In fact, legacy insurance might even harm customers.

While we have conducted extensive theoretical analysis of the automated mobility market, there remains substantial work in order to design and effect policies in the automated mobility market. For example, we have assumed the existence of indices z^M and z^I that link safety and manufacturing costs. However, how to build these indices from accident data and existing technology is unclear. Moreover, the information structure–who knows what about who–is also unclear and has not been accounted for here. Our further work will explore these areas.

CRediT authorship contribution statement

Daniel Vignon: Conceptualization, Methodology, Validation, Formal analysis, Investigation; Writing – original draft, review & editing. **Sina Bahrami:** Methodology, Numerical analysis, Visualization; Writing – review & editing.

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Appendices

Appendix A Derivation of optimality conditions

A.1 First-best

Let $\hat{\tau} = \sum_{n\in\{M,I,L\}}(\tau^n + \sum_{j\in\{r,s\}}\phi_1^{j,n})$ and $\hat{\phi}^j = \sum_{n\in\{M,I,L\}}\phi_2^{j,n}$. The FONC with respect to $\hat{\tau}$ and $\hat{\phi}^j$ yield:

$$\lambda + \lambda^0 \cdot (\hat{\tau}^M - c^M - \sum_{j\in\{r,s\}} \tilde{p}^j \cdot \phi_2^{j,M}) \cdot \Lambda' = -\lambda \cdot \frac{1}{\frac{\partial V}{\partial \hat{\tau}}}$$

$$\lambda + \lambda^0 \cdot (\hat{\tau}^M - c^M - \sum_{j\in\{r,s\}} \tilde{p}^j \cdot \phi_2^{j,M}) \cdot \Lambda' = \lambda \cdot \frac{\tilde{p}^j}{\frac{\partial V}{\partial \hat{\phi}^j}} \quad \forall j \in \{r,s\}$$

It follows from the above that:

$$\begin{aligned} &-\frac{1}{\frac{\partial V}{\partial \hat{\tau}}} = \frac{\tilde{p}^r}{\frac{\partial V}{\partial \hat{\phi}^r}} = \frac{\tilde{p}^s}{\frac{\partial V}{\partial \hat{\phi}^s}} \\ &\implies U'(W^r) = U'(W^s) \end{aligned} \tag{16}$$

Now, $U'(\cdot)$ is strictly decreasing as a consequence of our DARA assumption. Thus, it follows that $W^r = W^s$ which implies that $\hat{\phi}^r - d^r = \hat{\phi}^s - d^s$. Moreover:

$$\begin{aligned} &-\frac{\partial V}{\partial \hat{\tau}} = (1 - \tilde{p}^r - \tilde{p}^s) \cdot U'(W^0) + (\tilde{p}^r + \tilde{p}^s) \cdot U'(W^r) \\ &\implies U'(W^r) = (1 - \tilde{p}^r - \tilde{p}^s) \cdot U'(W^0) + (\tilde{p}^r + \tilde{p}^s) \cdot U'(W^r) \\ &\implies U'(W^0) = U'(W^r) = U'(W^s) \end{aligned}$$

As a consequence, $-\frac{\partial V}{\partial \hat{\tau}} = U'(W^0)$ and $\hat{\phi}^j = d^j \quad \forall j \in \{r,s\}$. These steps provide the essential ingredients to deriving Equations (2a) and (2b).

Moreover, as a consequence of $U'(W^0) = U'(W^r) = U'(W^s)$, $\frac{\partial V}{\partial z^M} = \frac{\partial V}{\partial z^I} = 0$ which is the key ingredient to deriving Equations (2c) and (2d).

A.2 Monopoly and Nash game

The steps to derive the FONC equations are the same as in the first-best case and identical wealth in every state readily obtains.

A.3 Liability rule optimality

After forming and taking the derivatives of the Lagrangians for (N-MR) and (N-IR), we obtain the following:

$$-C^{M'}(z^{M,*}) = \frac{\partial \tilde{p}^r}{\partial z^M}(z^{M,*}, z^{I,*}) \cdot d^r + \frac{\partial \tilde{p}^s}{\partial z^M}(z^{M,*}, z^{I,*}) \cdot \phi^{s,M} - \mu^M \cdot \mathrm{A}_1\left(z^{M,**} - z^{M,*}, z^{I,**} - z^{I,*}\right) \cdot d^s \tag{17a}$$

$$-C^{I'}(z^{I,*}) \cdot \eta^s \cdot l = \frac{\partial \tilde{p}^s}{\partial z^I}(z^{M,*}, z^{I,*}) \cdot \phi^{s,I} \cdot \lambda^* + \mu^I \cdot \mathrm{A}_2\left(z^{M,**} - z^M, z^{I,**} - z^{I,*}\right) \cdot d^s \cdot \lambda^* \tag{17b}$$

Now, by definition, $\phi^{s,M} = \alpha \cdot d^s$ and $\phi^{s,I} = (1-\alpha) \cdot d^s$. Additionally, at the first-best, we have:

$$-C^{M'}(z^{M,**}) = \frac{\partial \tilde{p}^r}{\partial z^M}(z^{M,**}, z^{I,**}) \cdot d^r + \frac{\partial \tilde{p}^s}{\partial z^M}(z^{M,**}, z^{I,**}) \cdot d^s \tag{18a}$$

$$-C^{I'}(z^{I,**}) \cdot \eta^s \cdot l = \frac{\partial \tilde{p}^s}{\partial z^I}(z^{M,**}, z^{I,**}) \cdot d^s \cdot \lambda^{**} \tag{18b}$$

Thus, assuming that the rule reproduces the first-best technology levels (i.e. $z^{n,*} = z^{n,**}$ for $n \in \{M, I, L\}$), we can subtract Equation (18) from Equation (17) and the result derived in Section 3.4.1 readily follows.

Appendix B Heterogeneity in accident types and severity

In our model, we have considered that, on either portion of the road, only one type of accident is possible. However, in practice, accidents are of different types and come with their own level of severity. In this context, we are interested in a) our conclusions regarding optimal coverage and liability rules and b) the role of legacy insurers.

The fundamental assumption in this subsection is that the distribution of accidents is such that, for any possible accident, it is possible to draw up an insurance contract. This is equivalent to collective insurance coverage between automakers, ISSPs, and legacy insurers being such that it covers all possible realizations of accident damage.

To deal with the heterogeneity in accident types, we resort to the notion of ”pathways to harm” introduced in Talley (2019). We assume that there exist multiple pathways to harm denoted by $h \in \mathcal{H} \subset \mathbb{R}$. These pathways to harm are distributed according to a PDF $f(\cdot; l)$ with support $[\underline{h}, \overline{h}]$ and which parametrically depends on the distance travelled, l.

Additionally, each of these pathways is associated with a damage level d_h^j on road portion k. We assume that, all pathways to harm such that $h \leq t$ can be avoided, where $t = T(z^M, z^I)$ is the maximal pathway to harm that can be avoided with vehicle and infrastructure technology z^M and z^I.

Assumption 6. *An increase in road length decreases the probability that no accident occurs:* $\frac{d}{dl} \int_{\underline{h}}^{T(z^M, z^I)} f(h; l) \cdot dh < 0$.

In essence, driving longer distances increases the probability of getting into an accident, regardless of its type and of the technology level.

Given our framework, V becomes:

$$\begin{aligned} V = U(W^0) \cdot \prod_k \int_{\underline{h}}^{T(z^M, z^{I,k})} f\left(h; \eta^k \cdot l\right) \cdot dh + \int_{T(z^M,0)}^{\overline{h}} f\left(h; \eta^r \cdot l\right) \cdot U(W_h^r) \cdot dh + \\ \int_{\underline{h}}^{T(z^M,0)} f\left(h; \eta^r \cdot l\right) \cdot dh \cdot \int_{T(z^M,z^I)}^{\overline{h}} f\left(h; \eta^s \cdot l\right) \cdot U(W_h^s) \cdot dh \end{aligned} \tag{19a}$$

$$W_h^j = \tilde{W}^0 - \sum_{n \in \{M,I,L\}} (\tau^n + \sum_{j \in \{r,s\}} \phi_{1h}^{j,n} - \phi_{2h}^{j,n}) - d_h^j \quad \forall j \in \{r, s\} \tag{19b}$$

$$z^{I,k} = \begin{cases} 0 & \text{if } k = r \\ z^I & \text{if } k = s \end{cases} \tag{19c}$$

First, we note that, despite the introduction of different accident types and their related damages, the structure of the equation describing the value of car ownership, Equation (19a), is essentially left unchanged. Thus, we naturally expect that many of the insights derived in Sections 3.2 to 3.4 should carry over. Moreover, we also note that a key driver of the optimality of full coverage is that customers are identical in their risk profile and equalize marginal utility across states. Thus, the number of states should in itself be irrelevant. We show in Appendix B that, indeed, full coverage is still optimal in the first-best, the monopoly, and the Nash game. Additionally, in the case of the later, the introduction of a liability rule can still induce first-best automation and digitalization levels even in the absence of legacy insurers. Thus, heterogeneity in accident types does not induce a necessity for the presence of legacy insurers in the automated driving market.

References

J. Borenstein, J. R. Herkert, and K. W. Miller. Self-Driving Cars and Engineering Ethics: The Need for a System Level Analysis. *Science and Engineering Ethics*, 25(2):383–398, Apr. 2019. ISSN 1471-5546. doi: 10.1007/s11948-017-0006-0. URL https://doi.org/10.1007/s11948-017-0006-0.

Cambridge Systematics. Crashes vs Congestion: What's the Cost to Society. Technical report, AAA, Mar. 2008. URL http://virtualadjacency.com/wp-content/uploads/2008/01/33b-2-Crashes-vs-Congestion-AAA-report-Mar08-Full.pdf.

I. Chatterjee and G. A. Davis. Evolutionary Game Theoretic Approach to Rear-End Events on Congested Freeway. *Transportation Research Record*, 2386(1):121–127, Jan. 2013. ISSN 0361-1981. doi: 10.3141/2386-14. URL https://doi.org/10.3141/2386-14. Publisher: SAGE Publications Inc.

N. W. Communications. When a Tesla on Autopilot Kills Someone, Who Is Responsible?, Mar. 2022. URL http://www.nyu.edu/content/nyu/en/about/news-publications/news/2022/march/when-a-tesla-on-autopilot-kills-someone--who-is-responsible--.

G. Contissa, F. Lagioia, and G. Sartor. The Ethical Knob: ethically-customisable automated vehicles and the law. *Artificial Intelligence and Law*, 25(3):365–378, Sept. 2017. ISSN 1572-8382. doi: 10.1007/s10506-017-9211-z. URL https://doi.org/10.1007/s10506-017-9211-z.

P. J. Cook and D. A. Graham. The Demand for Insurance and Protection: The Case of Irreplaceable Commodities. *The Quarterly Journal of Economics*, 91(1):143–156, 1977. ISSN 0033-5533. doi: 10.2307/1883142. URL https://www.jstor.org/stable/1883142. Publisher: Oxford University Press.

A. F. Daughety and J. F. Reinganum. Economic analysis of products liability: Theory. *Research Handbook on the Economics of Torts*, pages 69–96, Nov. 2013. URL https://www.elgaronline.com/display/edcoll/9781848441187/9781848441187.00011.xml. ISBN: 9781781006177 Publisher: Edward Elgar Publishing Section: Research Handbook on the Economics of Torts.

H. Dawid and G. Muehlheusser. Smart products: Liability, investments in product safety, and the timing of market introduction. *Journal of Economic Dynamics and Control*, 134:104288, Jan. 2022. ISSN 0165-1889. doi: 10.1016/j.jedc.2021.104288. URL https://www.sciencedirect.com/science/article/pii/S0165188921002232.

H. Dawid, X. Di, P. M. Kort, and G. Muehlheusser. Autonomous vehicles policy and safety investment: an equilibrium analysis with endogenous demand, 2023. URL https://papers.ssrn.com/abstract=4409161.

X. Di, X. Chen, and E. Talley. Liability design for autonomous vehicles and human-driven vehicles: A hierarchical game-theoretic approach. *Transportation Research Part C: Emerging Technologies*, 118:102710, Sept. 2020. ISSN 0968-090X. doi: 10.1016/j.trc.2020.102710. URL https://www.sciencedirect.com/science/article/pii/S0968090X20306252.

G. Dionne. Moral Hazard and State-Dependent Utility Function. *The Journal of Risk and Insurance*, 49(3):405–422, 1982. ISSN 0022-4367. doi: 10.2307/252495. URL https://www.jstor.org/stable/252495. Publisher: [American Risk and Insurance Association, Wiley].

I. Ehrlich and G. S. Becker. Market Insurance, Self-Insurance, and Self-Protection. *Journal of Political Economy*, 80(4):623–648, 1972. ISSN 0022-3808. URL https://www.jstor.org/stable/1829358. Publisher: University of Chicago Press.

J. Elias. Taking a driverless Waymo in Phoenix over the holidays was fun but unsettling, Jan. 2022. URL https://www.cnbc.com/2022/01/08/heres-what-it-was-like-to-ride-in-a-waymo-with-no-driver-in-phoenix.html. Section: Technology.

R. Elliott. WSJ News Exclusive | Congressmen, Consumer Reports Raise Concerns Over Tesla's Autopilot. *Wall Street Journal*, Apr. 2021. ISSN 0099-9660. URL https://www.wsj.com/articles/tesla-crashes-raise-questions-in-congress-over-potential-pattern-of-safety-concerns-1161910

R. Elliott and R. Felton. Tesla Prevails in One of First Trials Over Autopilot's Role in a Fatal Crash. *WSJ*, Oct. 2023. URL https://www.wsj.com/business/autos/tesla-prevails-in-one-of-first-trials-over-autopilots-role-in-a-fatal-crash-33582ece. Section: Business.

R. Elvik. A review of game-theoretic models of road user behaviour. *Accident Analysis & Prevention*, 62:388–396, Jan. 2014. ISSN 0001-4575. doi: 10.1016/j.aap.2013.06.016. URL https://www.sciencedirect.com/science/article/pii/S0001457513002443.

D. J. Fagnant and K. Kockelman. Preparing a nation for autonomous vehicles: opportunities, barriers and policy recommendations. *Transportation Research Part A: Policy and Practice*, 77:167–181, July 2015. ISSN 0965-8564. doi: 10.1016/j.tra.2015.04.003. URL https://www.sciencedirect.com/science/article/pii/S0965856415000804.

E. Feess and G. Muehlheusser. Autonomous Vehicles: Moral Dilemmas and Adoption Incentives. *SSRN Electronic Journal*, 2022. ISSN 1556-5068. doi: 10.2139/ssrn.4158130. URL https://www.ssrn.com/abstract=4158130.

FHWA. CARMA Program Overview, Apr. 2021. URL https://highways.dot.gov/research/operations/CARMA.

M. Grinell. Michigan, Cavnue Creating Road of Future Between Ann Arbor and Detroit | Michigan Business, Aug. 2020. URL https://www.michiganbusiness.org/press-releases/2020/08/michigan-cavnue-creating-road-of-future-between-ann-arbor-and-detroit/.

J. Hertz. The Potential of "Smart Roads:" Integrated Roadways Reimagines Infrastructure - News, Mar. 2022. URL https://www.allaboutcircuits.com/news/the-potential-of-smart-roads-integrated-roadways-reimagines-infrastructure/.

J. Himmelreich. Never Mind the Trolley: The Ethics of Autonomous Vehicles in Mundane Situations. *Ethical Theory and Moral Practice*, 21(3):669–684, June 2018. ISSN 1572-8447. doi: 10.1007/s10677-018-9896-4. URL https://doi.org/10.1007/s10677-018-9896-4.

Y. Jin and S. Vasserman. Buying Data from Consumers: The Impact of Monitoring Programs in U.S. Auto Insurance. Working Paper 29096, National Bureau of Economic Research, July 2021. URL https://www.nber.org/papers/w29096. Series: Working Paper Series.

W. M. Landes and R. A. Posner. A Positive Economic Analysis of Products Liability. *The Journal of Legal Studies*, 14(3):535–567, Dec. 1985. ISSN 0047-2530, 1537-5366. doi: 10.1086/467785. URL https://www.journals.uchicago.edu/doi/10.1086/467785.

J. A. Ligon and P. D. Thistle. Information Asymmetries and Informational Incentives in Monopolistic Insurance Markets. *The Journal of Risk and Insurance*, 63(3):434–459, 1996. ISSN 0022-4367. doi: 10.2307/253620. URL https://www.jstor.org/stable/253620. Publisher: [American Risk and Insurance Association, Wiley].

M. F. Lohmann. Liability Issues concerning Self-Driving Vehicles Special Issue on the Man and the Machine. *European Journal of Risk Regulation (EJRR)*, 7(2):335–340, 2016. URL https://heinonline.org/HOL/P?h=hein.journals/ejrr2016&i=355.

S. Nyholm. The ethics of crashes with self-driving cars: A roadmap, II. *Philosophy Compass*, 13(7):e12506, 2018. ISSN 1747-9991. doi: 10.1111/phc3.12506. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/phc3.12506. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/phc3.12506.

S. Nyholm and J. Smids. The Ethics of Accident-Algorithms for Self-Driving Cars: an Applied Trolley Problem? *Ethical Theory and Moral Practice*, 19(5):1275–1289, Nov. 2016. ISSN 1572-8447. doi: 10.1007/s10677-016-9745-2. URL https://doi.org/10.1007/s10677-016-9745-2.

W. Y. Oi. The Economics of Product Safety. *The Bell Journal of Economics and Management Science*, 4(1):3–28, 1973. URL https://www.jstor.org/stable/3003137.

J. A. Ordover. Products Liability in Markets with Heterogeneous Consumers. *The Journal of Legal Studies*, 8(3):505–525, June 1979. ISSN 0047-2530, 1537-5366. doi: 10.1086/467619. URL https://www.journals.uchicago.edu/doi/10.1086/467619.

M. V. Pauly. Overinsurance and Public Provision of Insurance: The Roles of Moral Hazard and Adverse Selection. *The Quarterly Journal of Economics*, 88(1):44, Feb. 1974. ISSN 00335533. doi: 10.2307/1881793. URL https://academic.oup.com/qje/article-lookup/doi/10.2307/1881793.

A. M. Polinsky. Strict Liability vs. Negligence in a Market Setting. *The American Economic Review*, 70(2):363–367, 1980. ISSN 0002-8282. URL https://www.jstor.org/stable/1815499. Publisher: American Economic Association.

R. A. Posner. A Theory of Negligence. *The Journal of Legal Studies*, 1(1):29–96, 1972. ISSN 0047-2530. URL https://www.jstor.org/stable/724011. Publisher: [University of Chicago Press, University of Chicago Law School].

R. A. Posner and W. M. Landes. The Positive Economic Theory of Tort Law. *Georgia Law Review*, 15:851–924, 1980.

M. Rothschild and J. Stiglitz. Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. *The Quarterly Journal of Economics*, 90(4):629, Nov. 1976. ISSN 00335533. doi: 10.2307/1885326. URL https://academic.oup.com/qje/article-lookup/doi/10.2307/1885326.

H. Schlesinger. Nonlinear Pricing Strategies for Competitive and Monopolistic Insurance Markets. *The Journal of Risk and Insurance*, 50(1):61–83, 1983. ISSN 0022-4367. doi: 10.2307/252417. URL https://www.jstor.org/stable/252417. Publisher: [American Risk and Insurance Association, Wiley].

A. Shariff, J.-F. Bonnefon, and I. Rahwan. How safe is safe enough? Psychological mechanisms underlying extreme safety demands for self-driving cars. *Transportation Research Part C: Emerging Technologies*, 126:103069, May 2021. ISSN 0968-090X. doi: 10.1016/j.trc.2021.103069. URL https://www.sciencedirect.com/science/article/pii/S0968090X21000942.

I. R. a. A. Shariff. Self-Driving Cars Could Save Many Lives. But Mental Roadblocks Stand in the Way. *Wall Street Journal*, Apr. 2021. ISSN 0099-9660. URL https://www.wsj.com/articles/self-driving-cars-could-save-many-lives-but-mental-roadblocks-stand-in-the-way-11617732000.

S. Shavell. Strict Liability versus Negligence. *The Journal of Legal Studies*, 9(1):1–25, 1980. ISSN 0047-2530. URL https://www.jstor.org/stable/724036. Publisher: [University of Chicago Press, University of Chicago Law School].

S. Shavell. On the Redesign of Accident Liability for the World of Autonomous Vehicles. *The Journal of Legal Studies*, 49(2):243–285, June 2020. ISSN 0047-2530. doi: 10.1086/711027. URL https://www.journals.uchicago.edu/doi/full/10.1086/711027. Publisher: The University of Chicago Press.

J. E. Stiglitz. Monopoly, Non-Linear Pricing and Imperfect Information: The Insurance Market. *The Review of Economic Studies*, 44(3):407–430, 1977. ISSN 0034-6527. doi: 10.2307/2296899. URL https://www.jstor.org/stable/2296899. Publisher: [Oxford University Press, Review of Economic Studies, Ltd.].

E. Talley. Automatorts: How Should Accident Law Adapt to Autonomous Vehicles? Lessons from Law and Economics. page 37, Jan. 2019.

S. M. Thornton, S. Pan, S. M. Erlien, and J. C. Gerdes. Incorporating Ethical Considerations Into Automated Vehicle Control. *IEEE Transactions on Intelligent Transportation Systems*, 18(6):1429–1439, June 2017. ISSN 1558-0016. doi: 10.1109/TITS.2016.2609339. Conference Name: IEEE Transactions on Intelligent Transportation Systems.

Tucker, Sean. Mercedes: We'll Be Liable for Self-Driving Cars, Mar. 2022. URL https://www.kbb.com/car-news/mercedes-well-be-liable-for-self-driving-cars/.

E. T. Verhoef and J. Rouwendal. Pricing, Capacity Choice, and Financing in Transportation Networks. *Journal of Regional Science*, 44(3):405–435, 2004. ISSN 1467-9787. doi: https://doi.org/10.1111/j.0022-4146.2004.00343.x. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.0022-4146.2004.00343.x. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.0022-4146.2004.00343.x.

D. A. Vignon, Y. Yin, S. Bahrami, and K. Laberteaux. Economic analysis of vehicle infrastructure cooperation for driving automation. *Transportation Research Part C: Emerging Technologies*, 142: 103757, Sept. 2022. ISSN 0968090X. doi: 10.1016/j.trc.2022.103757. URL https://linkinghub.elsevier.com/retrieve/pii/S0968090X22001905.

S. S. Wu. Autonomous vehicles, trolley problems, and the law. *Ethics and Information Technology*, 22(1):1–13, Mar. 2020. ISSN 1572-8439. doi: 10.1007/s10676-019-09506-1. URL https://doi.org/10.1007/s10676-019-09506-1.