

## Abstract

This study proposes a new safety metric to estimate the driving risks of vehicles and road sections on the basis of vehicle risk field model and the concept of power of the work done by the field force (PWF). The proposed metric based on PWF is not only able to estimate the risk associated with specific vehicle interactions like previous studies, but can also be aggregated to capture the overall risk imposed by the entire surrounding traffic on a vehicle and quantify the total risk on a specific road section. Case studies involving three different scenarios compared to time-to-collision (TTC) were conducted to validate the properties of the proposed PWF-based metric and demonstrate its effectiveness in assessing driving risks of vehicles in chaotic traffic and quantifying the overall risks for an entire road section.

## Modeling Vehicle Risk Field

### 1. Field Strength (Based on Tan et al.)

$$\text{Field Strength (vector)} \quad \mathbf{E}_{ij} = -|\mathbf{E}_{ij}| \cdot \frac{\nabla \mathbf{E}_{ij}}{|\nabla \mathbf{E}_{ij}|}$$

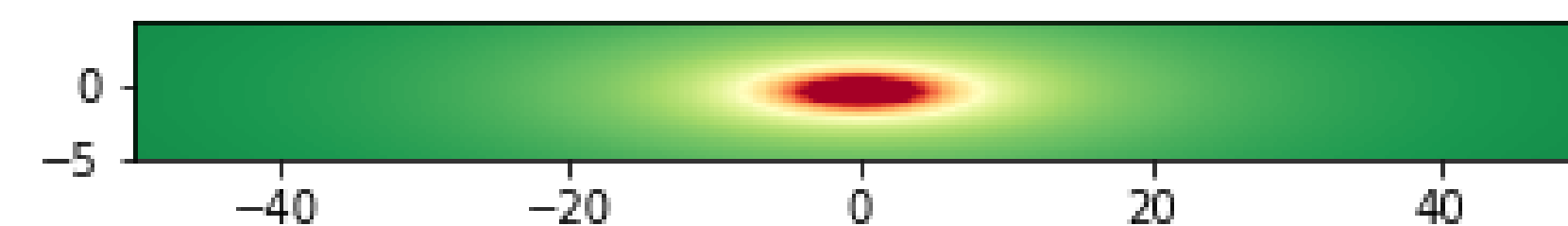
$$\text{Field Strength Magnitude} \quad |\mathbf{E}_{ij}| = \frac{1}{\sqrt{\delta_x^2 + \delta_y^2 + 1}}$$

$\delta_x$  and  $\delta_y$  are decay factors along x and y axis, such that

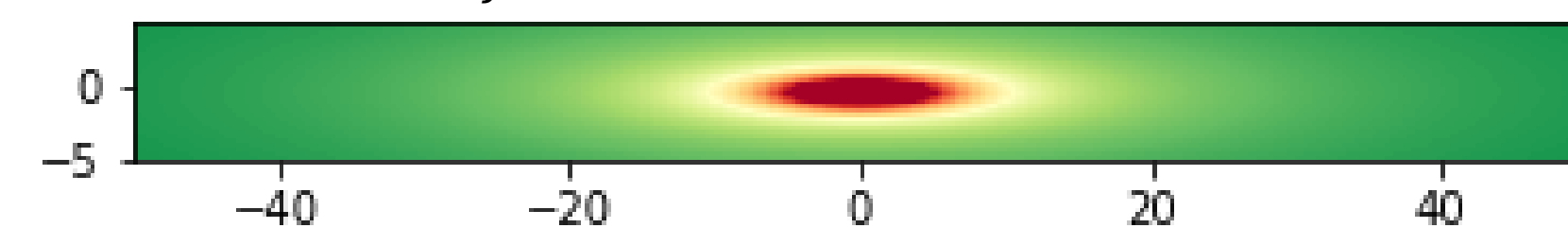
$$\delta_x = \frac{\beta_{i,x}[(x_j - x_i)\cos\theta_i + (y_j - y_i)\sin\theta_i]}{\alpha_{i,x}|\mathbf{v}_i| + 1}$$

$$\delta_y = \frac{\beta_{i,y}[(y_j - y_i)\cos\theta_i + (x_j - x_i)\sin\theta_i]}{\alpha_{i,y}|\mathbf{v}_i| + 1}$$

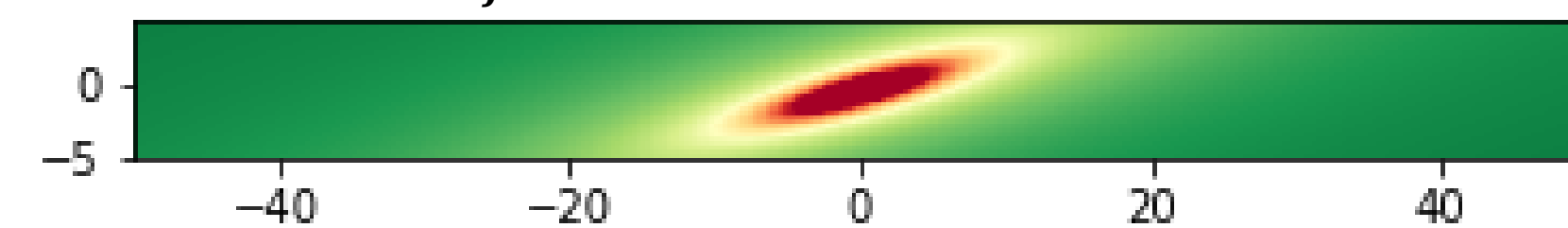
$\mathbf{v}_i$  and  $\theta_i$  are the velocity vector and heading of vehicle i



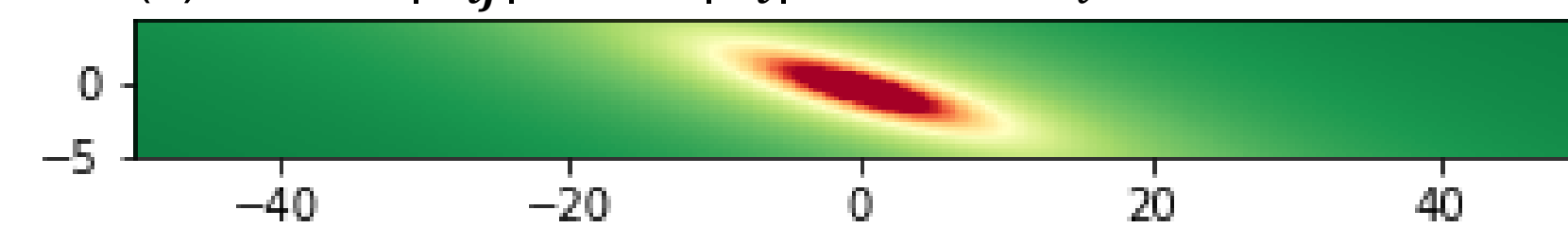
(a) Plot of  $|\mathbf{E}_{ij}|$  when  $|\mathbf{v}_i|=0$  m/s,  $\theta_i = 0^\circ$



(b) Plot of  $|\mathbf{E}_{ij}|$  when  $|\mathbf{v}_i|=40$  m/s,  $\theta_i = 0^\circ$



(c) Plot of  $|\mathbf{E}_{ij}|$  when  $|\mathbf{v}_i|=40$  m/s,  $\theta_i = 15^\circ$



(d) Plot of  $|\mathbf{E}_{ij}|$  when  $|\mathbf{v}_i|=40$  m/s,  $\theta_i = -15^\circ$

### 2. Risk Assessment Metrics

#### Vehicle-to-vehicle level risk assessment

$$PWF_{ij} = \mathbf{F}_{ij} \cdot (\mathbf{v}_i - \mathbf{v}_j) = M_j \mathbf{E}_{ij} \cdot (\mathbf{v}_i - \mathbf{v}_j)$$

$PWF_{ij}$  denotes the *power of field force* vehicle i imposed on vehicle j, which is the increment of potential energy of vehicle j with respect to vehicle i.  $\mathbf{F}_{ij}$  denotes the field force of vehicle i imposed on vehicle j.  $M_j = m_j(1.566 \times 10^{-14}|\mathbf{v}_j|^{6.687} + 0.3345)$  (Wang et al.)

#### Environment-to-vehicle level risk assessment

$$PWF_{ij}^+ = \sum_{i \neq j \in V_j} w_{ij} \cdot PWF_{ij}^+ = \sum_{i \neq j \in V_j} w_{ij} \cdot PWF_{ij} \cdot \mathbf{1}_{PWF_{ij} > 0} \quad PWF_{ij}^- = \sum_{i \neq j \in V_j} w_{ij} \cdot PWF_{ij}^- = \sum_{i \neq j \in V_j} w_{ij} \cdot PWF_{ij} \cdot \mathbf{1}_{PWF_{ij} < 0}$$

$V_j$  is a collection of vehicles in the proximity of vehicle j.  $w_{ij} = \min\left\{1, \left(\frac{2L}{l_{ij}}\right)^d\right\}$  is a weight factor, and  $l_{ij} = (x_i - x_j)\cos\theta_i - (y_i - y_j)\sin\theta_i$ .

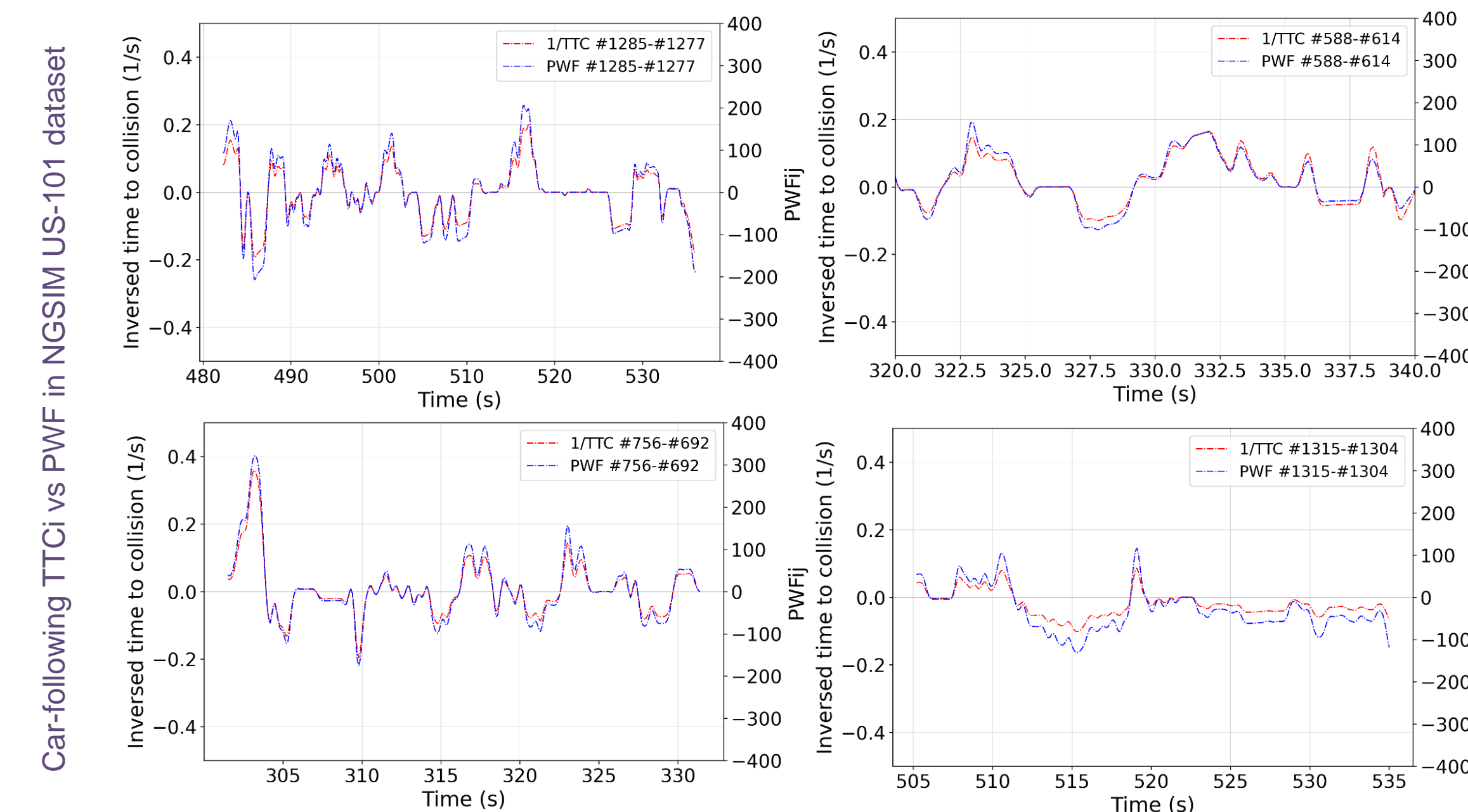
#### Sectional risk assessment

$$PWF_S^+ = \sum_{j \in V_S} PWF_j^+ \quad PWF_S^- = \sum_{j \in V_S} PWF_j^- \quad V_S \text{ denotes the collection of all vehicles on section } S.$$

## Results and Takeaways

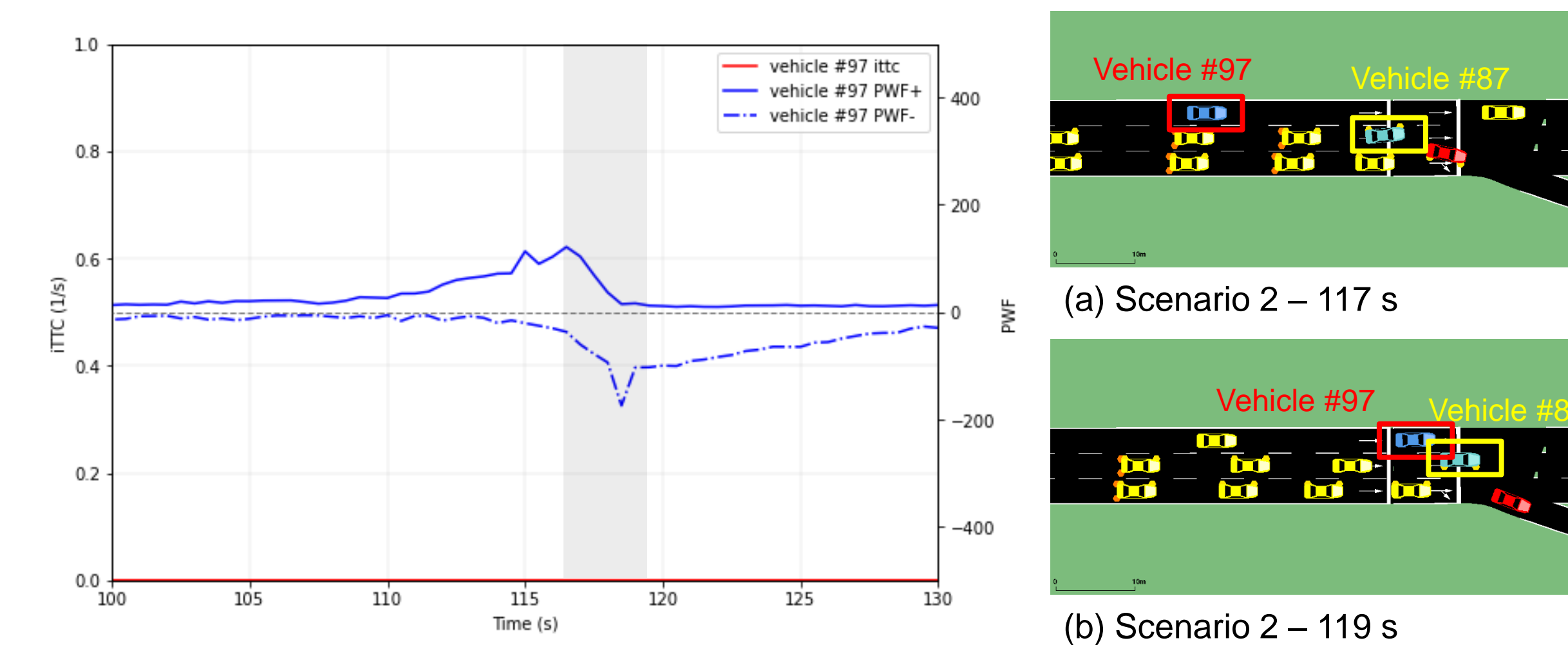
### 1. Scenario 1: NGSIM Car-following

Vehicle-to-vehicle PWF shows the same trend as inverse TTC (TTCi) in car-following scenarios in NGSIM US-101 trajectory dataset



### 2. Scenario 2: Simulated Stop and Merging

In a SUMO simulation, a 3-lane highway with an off-ramp operated at 75% capacity. A vehicle, inattentive to its exit, stopped 10 meters before the off-ramp, waiting 5 seconds to merge right and exit. This caused subsequent vehicles to brake or swerve to avoid a collision.



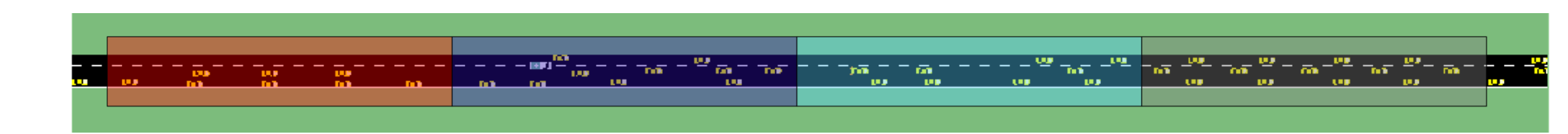
Vehicle #97 TTCi vs environment-to-vehicle level PWF

Vehicle #97 faced collision risk from vehicle #87's sudden lane change, a scenario not foreseen by TTC but indicated by a PWF+ peak at the environment-to-vehicle level.

- PWF+ outperforms TTC by capturing side collision risks without trajectory intersections.
- PWF+ identifies risks from significant speed differences between lanes. PWF- reflects risk dissipation intensity, with high values indicating evasive actions or relief from traffic congestion.

### 3. Scenario 3: Simulated Emergency Vehicle Passing

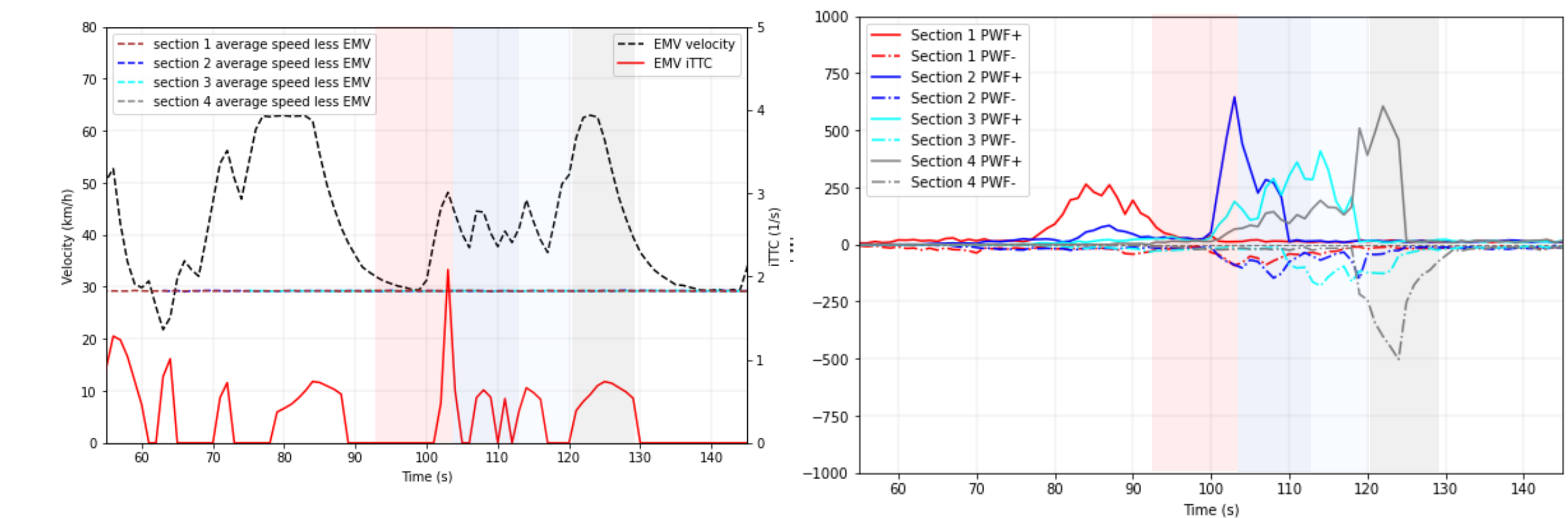
Scenario 3 confirmed the effectiveness of section-level PWF in identifying risks along four 100-meter road sections, each marked in different colors. PWF+ and PWF- changes were tracked as an Emergency Medical Vehicle (EMV) passed through these sections. Data on overtaking and yielding events during the EMV's passage, along with the EMV's speed profile, iTTC, and average traffic speed in each section, were also analyzed and recorded.



Scenario 3 simulates an EMV passing through four 100-meter sections

	Time frame when EMV was within the section (sec)	Number of vehicles EMV overtook	Number of yielding events during EMV passing through
Section 1	93-103	0	0
Section 2	103-112	2	3
Section 3	112-120	1	1
Section 4	120-129	3	3

Summary of overtaking and yielding events when EMV passed through each section



Speed profiles of the four sections and EMV and EMV TTCi Section-level PWF+ and PWF- of the four sections

- Section 1 PWF+ peaked due to speed difference while EMV approaching the section.
- Section 2 and 4 recorded relatively high PWF+ and TTCi due to interactions.
- Section 3 PWF+ peak was lower compared to section 2 because of fewer interactions.
- Section 4 showed relatively high PWF- peak as the EMV left the road.

### 4. Takeaways

- PWF can be aligned with TTCi in car-following scenarios.
- Vehicle-based PWF can detect side collision risks and speed discrepancies missed by TTC.
- Sectional PWF is efficient in summarizing interaction risks in a highway scenario with an overtaking emergency vehicle.