Naturalistic Driving Data Analytics: Safety Evaluation With Multi-state Survival Models*

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Abstract— **Naturalistic driving data analysis offers insights into risky driving behaviors at the trajectory level, which are critical to traffic safety. However, few studies discuss the modeling challenges of vehicle interactions that are multi-state and recurrent. In addition, escalation and de-escalation transitions are two competing events by nature, requiring extra care in statistical modeling. We propose Markov renewal survival models along with cause-specific and cumulative incidence function approaches for such trajectory analysis. This study aims to quantify transition hazards and predict duration to assess the impact of off-ramps on driving behaviors at 2 highway segments in Germany. We use non-parametric, semi-parametric, and parametric estimations and select the best-fitted models based on the corrected Akaike Information Criterion (AICc). The results show that off-ramps significantly increase de-escalation durations by 27% during risky states, while vehicle types show statistically significant impacts on escalation transitions as well. Furthermore, we discuss the limitations of the cause-specific approach and recommend the use of the cumulative incidence function for predicting the marginal survival function in the presence of competing events.**

I. INTRODUCTION

Traffic safety evaluation at specific sites requires years of data collection since crash events are rare. Such a long data collection process can be reactive regarding that the crashes have already occurred without prevention strategies being adjusted in a timely manner. Additionally, safety evaluations based on crash reports may fail to reveal the underlying causes of crashes when subject to biases such as "not-cause-if-normal". For example, the police put an effort into investigating whether the crash-involved drivers are compliant with traffic regulations or not, namely, speeding. However, it's usually not their responsibility to question whether the speed limit itself is set reasonably in the first place [1]. One consequence is the failure to properly evaluate the impact of ill-designed infrastructures on driving behaviors which results in a systematic risk of accidents. With computer vision technologies, analyzing naturalistic driving data becomes available to evaluate road segments proactively. Different from traditional safety analysis that reactively waits for crashes to occur, the objective of proactive safety analysis focuses on evaluating near-crash events identified by safety surrogate measures at a granular level [2, 3]. According to the concept in [4], the proportions of crashes, near-crashes, potential conflicts, and undisturbed events form a "safety pyramid" structure. As near-crash events are more frequent and correlated to crash events, preventing the more frequent crash conflicts from escalating to crashes becomes intuitive and theoretically feasible to enhance traffic safety [5].

Survival analysis is a widely adopted proactive safety evaluation approach for its advantage of quantifying hazards and associating discrete events with aggregated covariates [6]. Recent papers mostly used single transition survival models [7]–[9], disregarding the recurrent multi-state driving behaviors in naturalistic driving settings. One method to estimate the hazard functions of recurrent events is to use the Markov renewal models, which relaxes the Markovian assumptions by assuming the sojourn times between two states are independent distributions and only depend on the recent entering state [10, 11]. A cause-specific Markov renewal model was applied to study road user behaviors with recurrent events at a semi-controlled crosswalk in [12]. However, naturalistic driving modelings can be complicated due to competing events.

Given an interaction pair at a risky state, the transition from risky to near-crash (escalation) and from risky to safety states (de-escalation) are two mutually exclusive events, i.e., only one transition can occur at a time, such two events are considered as competing events. Modeling the two competing events requires caution as censoring the competing events may violate the underlying assumptions of survival models. There are two common modeling approaches for competing risks survival analysis: cause-specific (C-S) hazard and cumulative-hazard function (CIF) [12]–[15]. Few proactive safety analyses, however, performed safety evaluations considering the recurrent multi-states with the presence of competing risks.

The objective of this paper is to quantify the impact of infrastructure on driving behaviors considering recurrent and multi-state competing events in a multi-state survival modeling context using naturalistic driving data. We will first discuss two Markov renewal multi-state approaches: cause-specific and cumulative incidence function in section II. Two road segments, one that has an off-ramp, and the other without will be described in section III. We further discuss the advantages and disadvantages of the two approaches based on empirical results in section IV, with conclusions summarized in section V.

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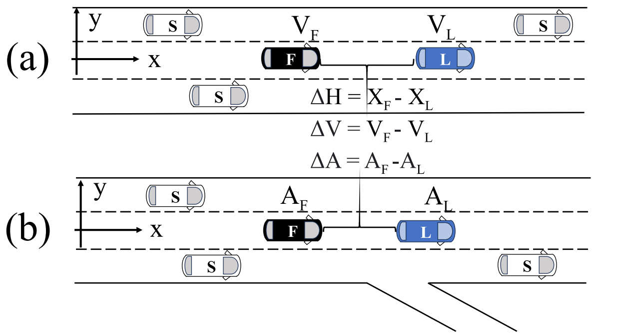

Fig. 1. Car-following state classification at a highway segment using MTTC

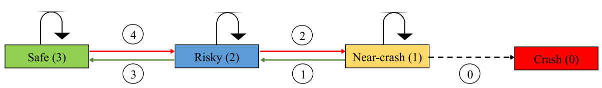

Fig. 2. State Transitions

II. Methodologies

A. Setting

Fig.1 shows the setting of a highway driving scenario. We use modified time-to-collisions (MTTC) to classify vehicle safety states, as it incorporates the variation of acceleration in highway [16]. ΔH, ΔV, ΔA in (1) represents the difference between the following vehicle and leading vehicles in headway, speed, and acceleration.

$$MTTC = \frac{(\Delta V) \pm \sqrt{\Delta V^2 + 2\Delta H \Delta A}}{\Delta A} \tag{1}$$

Denote the states of vehicle interaction pair i as $S \in \{0,1,2,3\}$ to mimic the 4 states of the safety pyramid as illustrated in Fig. 2. Let $k \in \{0,1,2,3,4\}$ represent the 5 transitions among these 4 states. Since crash states are rarely observed in naturalistic driving data, transition 0 will not be covered in this case study. In the context of Markov renewal model, transitions 1 and 4 have a single end state, which can be modeled as a simple survival model, while transitions 2 and 3 start from the same state, and end in different states, forming a competing relationship.

Let T denote the continuous time to experience the state transitions or censoring, and X is the sojourn time between two consecutive states. $\delta = 0$ represents the transition to the ending state is observed, $\delta = 1$ right censored. Denote the probability density function as $f(t)$, the cumulative distribution $F(t)$, and the survival function is defined as $S(t)$, which is $Pr(T \leq t)$. By definition, we have

$$S(t) = 1 - Pr(T \leq t) = 1 - F(t) = 1 - \int_0^t f(u)du \tag{2}$$

For the convenience of notation, we will use one subscript $\{k\}$ to represent a single transition event in II-B, and $\{s,e\}$ to represent start and end states for the multi-state survival analysis in II-C. Let $\mathbf{Z}_i$ represent a vector of covariates for the interaction i, which can be a time-invariant variable such as vehicle type or a time-variant variable such as speed.

B. Simple Survival Analysis

Simple survival analysis estimates the covariate effect on a transition with a single ending state. In Fig. 2, the transition from safe to risky and near-crash to risky events are single transition events for the Markov renewal model. The hazard rate for transitions 1 and 4 is defined as,

$$\lambda_k(t) = \lim_{\delta t \to 0} \frac{Pr(t \leq T < t + \delta t | T \geq t)}{\delta t}, k = \{1,4\} \tag{3}$$

The survival functions can be written as follows (see [17] for a detailed explanation)

$$S_k(t) = e^{-\int_0^t \lambda_k(u)du} \tag{4}$$

Equations (2) - (4) combined show the relationship among transition hazard $\lambda_k(t)$, survival function $S_k(t)$, and probability density function $f_k(t)$ for transitions $k \in \{1,4\}$. Obtaining one function can infer the rest. A common non-parametric approach generally infers the survival function first using Kaplan-Meier (K-M) estimator as below:

$$\hat{S}_k(t) = \prod_{t_i <= t} (1 - \frac{d_i}{r_i}) \tag{5}$$

where d_i is the number of vehicle interaction pairs experienced the transition event of interest k, and r_i is the number of pairs at risk (uncensored interactions not experiencing the transition k yet). One important K-M assumption is that the censoring distributions are non-informative to event time. Violation of this assumption can result in a biased estimation [18]. Cox proportional hazard (Cox-PH), as a semi-parametric approach, models the proportional hazard function as

$$\lambda_k(t|\mathbf{Z}) = \lambda_{k0}(t) e^{\boldsymbol{\beta}^T \mathbf{Z}} \tag{6}$$

The parameter $\boldsymbol{\beta}$ and baseline hazard $\lambda_{k0}(t)$ can be estimated by maximizing the partial log-likelihood and profile likelihood [19, 20]. Cox-PH compares the relative risk through the hazard ratio and the model's proportional hazard functions can be checked by Schoenfeld's test [21]. The accelerated failure time model makes the parametric assumption on the error term. It models the transition duration X as

$$logX = \boldsymbol{\beta}^T \mathbf{Z} + \sigma \varepsilon \tag{7}$$

Depending on the distribution of ε, parametric distributions can be Generalized gamma distribution, Weibull, Log-normal, etc. The density distribution of X, if assumed to be a Generalized gamma distribution, has a density function written as

$$f(t) = \frac{\alpha(\boldsymbol{\beta}^T \mathbf{Z})^{\rho} t^{\rho} (\alpha - 1) e^{-(\boldsymbol{\beta}^T \mathbf{Z}) t^{\alpha}}}{\Gamma(\rho)} \tag{8}$$

when $\rho = 1$, it become a Weibull distribution; $\rho \to \infty$ it becomes a Log-normal distribution. Other types of parametric distributions such as Gompertz can be found in [17].

C. Multi-state Survival Analysis

Conditional on the risky state ($S=2$), the competing state space is denoted as $E \in \{1,3\}$, representing safe and near-crash. Similar to the simple survival analysis, each approach can be modeled using non-parametric, semi-parametric, and parametric estimation.

Cause-specific: For $e \in E$, cause-specific hazard function is defined as,

$$\lambda_{2,e}(x) = \lim_{\delta x \to 0} \frac{Pr(x \le X < x+\delta x, E=e|X \ge x)}{\delta x} \tag{9}$$

Its cumulative cause-specific hazard function is written as,

$$\Lambda_{2,e}(x) = \int_0^x \lambda_{2,e}(u)du \tag{10}$$

The survival function of the competing transition event from 2 to e is

$$S_{2,e}(x) = e^{-\Lambda_{2,e}(x)} \tag{11}$$

$S_{2,e}(x)$ is not the marginal distribution unless the two competing events are independent. Since escalation and de-esclation transitions at the risky state are competing events by nature, the survival function of risky events $S_2(x) = e^{-\sum_{j\in E}\Lambda_{2,e}(x)}$ is thus, smaller or equal to $S_{2,e}(s)$ (see [22] for a detailed explanation.) Cause-specific non-parametric estimation assuming the competing event as censored is similar to (5). The semi-parametric models of the cause-specific hazards can be modeled using Cox regression as shown in (6). As for parametric estimation, AFT models the observed durations $\tilde{X}_2$ as $min(X_{21}, X_{23})$, replacing the duration X in (7).

Cumulative Incidence Function: Due to the dependent nature of the escalation and de-escalation transitions, the marginal distribution becomes unidentifiable [23]. In order to estimate the marginal probability of competing events, Fine and Gray [14] proposed to model the Cox regression on the cumulative hazard function based on the subdistribution hazard $h_{2,e}(x)$:

$$\lim_{\delta x \to 0} \frac{Pr(x \le X < x+\delta x, E=e|(X \ge x) \cup (E \ne e, X < x))}{\delta x} \tag{12}$$

The subdistribution hazard function assumes that the occurrence of one event leads its competing event time to infinity. The cumulative subdistribution hazard function is written as,

$$H_{2,e}(x) = -\int_0^x h_{2,e}(u)du \tag{13}$$

The cumulative incidence function, also known as the crude incidence function or subdistribution function, is defined by

$$I_{2,e}(x) = Pr(X <= x, E=e) \tag{14}$$

The relationship between $I_{2,e}(x)$ and $\lambda_{2,e}(x)$ is

$$I_{2,e}(x) = \int_0^x \lambda_{2,j}(u) S_2(u)du \tag{15}$$

Expressing the cumulative incidence function by cumulative subdistribution hazard is given by,

$$I_{2,e}(x) = 1 - e^{-H_{2,e}(x)} \tag{16}$$

Unlike the C-S non-parametric estimation, the CIF non-parametric approach models the survival function by adjusting the competing events in the risk set, this estimator is called the Aalen-Johnsen estimator. The semi-parametric approach for the subdistribution cumulative incidence function $I_{2,e}(x)$ is known as the Fine-Gray model. The parametric AFT models the latent duration $X^*_{2,e} = \mathbb{1}(E=e) \times X_{2,e} + \mathbb{1}(E \ne e) \times \infty$, which is also called as the crude-risk AFT model [15].

D. Model Selection Criterion

In order to select a parsimonious regression model, we adopt the corrected Akaike information criterion (AIC_c) considering the potential small sample sizes N in near-crash states [24].

$$AIC_c = -2logLik + 2k \times \frac{N}{N-k-1} \tag{17}$$

where k represents the model degree of freedom.

III. Naturalistic Driving Datasets

We analyzed a highway naturalistic driving dataset collected by drone in Germany. The data quality has been validated for the purpose of safety evaluation [25, 26]. The dataset contains 110,500 vehicles and 44500 driven kilometers with a 25 Hz frame rate. Our objective is to assess the off-ramp effects on risky driving behaviors adjusted for other factors such as vehicle types, speed, and volume.

A. Scope

We focus on 2825 car-following from the 58th to 60th vehicle trajectory recordings since they are three-lane road segments containing with and without off-ramps. Data is collected around 9:00 to 9:30 AM on Wednesday. The average traffic density of 0.055 vehicles per meter. Few of the following vehicles ($< 0.1\%$) are congested ($< 16 m/s$). About 60 % are below the speed limit (33 m/s) with the off-ramps, and 76 % below the speed limit without off-ramps. The states are classified using (1) based on the cumulative distribution functions mapping between the surrogate measure (seconds) and percentiles (%). The selection of threshold requires engineering needs, we use the percentile-based threshold [27] in this case study. MTTC between 0 seconds and 20% are defined as “near-crash” states, 20% to 80% “risky”, and negative, greater than 80% or null values (no preceding vehicles) are “safe” states. This is to mimic the “safety pyramid” structure that “near-crash” events are rare [4]. A parsimonious model includes time-independent variables such as vehicle type (vehicle vs. truck), and off-ramp (w/ vs w/o). As for the time-dependent variables, we include average speed, average acceleration, and left and right vehicle counts of the following vehicle in state duration. The car and truck ratios of w/ and w/o off-ramps are 0.81 and 0.79, respectively. The minimum number of adjacent vehicles for the two comparison groups is 0 while w/ off-ramp has a maximum of 5 unique adjacent vehicles and 6 for w/o off-ramp group. The average speed acceleration distribution for off-ramps is visually left-shifted compared with no off-ramps

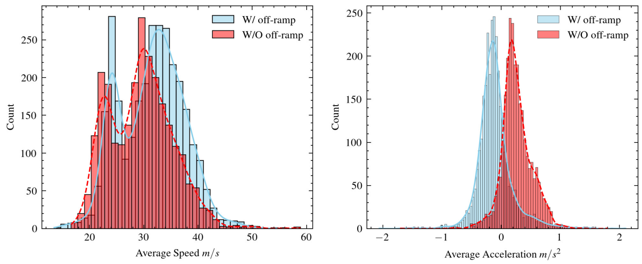

Fig. 3. Distribution of speed and acceleration of following vehicles

in Fig.3. How do these covariates affect each state transition will be answered in section IV.

B. Preprocessing

Since the existing function *msprep* in R package *mstate* is not applicable to recurrent transitions, we developed a preprocessing algorithm to extract the trajectories to match the "wide" formatting requirements from [28]. The main procedures are summarized in Alg. 1. To increase the model's robustness, continuous variables are discretized and factorized based on whether the speed is above the speed limit, acceleration is greater than 0 m/s^2 and, the nearby vehicles are present ($\geq$1). If the state remains unchanged throughout the observational time window, the ending states are censored by definition (see self-transition in Fig. 2). Additionally, the occurrence of competing events is treated as censored in the C-S modeling approach, while in the CIF approach, treated as the other factor kept in the risk sets [29].

Algorithm 1 Major steps for processing trajectory data

for vehicle interactions i =1,2,...,N **do**
    Calculate the MTTC of the car following interactions
    Classify the safety state of the following vehicles
    Transform the data frame into a wide format
    Merge the data with time-independent covariates
    Merge the data with time-dependent covariates
end for
Discretize and perform factorization on the covariates
Define transition matrix and censored events

IV. Multi-state trajectory analysis

Cause-specific and cumulative approaches are being implemented separately. We used *flexsurv* package in R [30] for C-S approach and *survival* package [29] for CIF approach. The best-fitted (semi-) parametric models are evaluated by choosing the smallest AIC_c values.

A. Model Identification

We deployed 4 parametric assumptions for the AFT model using the Weibull, Gompertz, generalized gamma, and Log-normal distribution. The baseline model is compared with the semi-parametric model (Cox-PH, Fine-Gray). We summarized the model selection metrics in Table I. The best-fitted AFT models for escalation transitions (2 and 4), are the generalized gamma distributions for both C-S and CIF approaches. As for de-escalation transitions (1 and 3), Log-normal distribution is found to be the best for the C-S approach.

Violation of the proportional hazard assumption is one of the major limitations of the Cox-PH model [31]. For the semi-parametric model, based on the p-values of the scaled Schoenfeld residual test, no proportional hazard assumption is violated except for the transition from a risky state in CIF approach, and the transition from the safe to the risky state.

B. Model Estimation

The estimated parameters for both approaches are summarized in Table II. For transitions 1 and 4, the C-S and CIF approaches provide the same parameter estimations, which is as expected as the lacking of competing events resulting in no difference in censorship. The challenging part is to interpret the results of the competing events when the estimations of the two approaches become inconsistent [32]. Cause-specific approaches are recommended for the purpose of understanding the impacts of covariates, regardless of the competing events [15, 33]. Conditional on risky states, the presence of an off-ramp can increase the de-escalation duration by a factor of 1.28 at the significance level of 0.05, adjusting for all other covariates. Based on the semi-parametric model, the cause-specific hazard ratio is 0.78, which represents that off-ramp reduces the de-escalation hazard rates given that the interaction pair does not experience censoring or competing

TABLE I
Semi-Parametric and Parametric Model Evaluation

Transition	Distribution	LogLik	AICc	p-value*
1	Cox-PH	-1324.4	2659.0	> 0.05
	Gompertz	-1400.1	2814.6	-
	Weibull	-1377.7	2769.8	-
	Generalized Gamma	-1362.6	2741.7	-
	Log-normal	-1362.7	2739.8	-
2 (C-S)	Cox-PH	-1918.5	3838.4	> 0.05
	Gompertz	-2019.5	4053.0	-
	Weibull	-2012.2	4038.6	-
	Generalized Gamma	-1990.1	3996.3	-
	Log-normal	-1997.2	4008.4	-
3 (C-S)	Cox-PH	-11736.9	23475.1	> 0.05
	Gompertz	-9353.4	18720.8	-
	Weibull	-9277.3	18568.8	-
	Generalized Gamma	-9172.9	18362.0	-
	Log-normal	-9173.7	18361.6	-
2 (CIF)	Fine-Gray	-2152.3	4314.7	$<$**0.05**
	Gompertz	-2260.3	4534.7	-
	Weibull	-2383.3	4780.7	-
	Generalized Gamma	-2257.4	4530.8	-
	Log-normal	-2359.9	4734.0	-
3 (CIF)	Fine-Gray	-12357.8	24725.6	$<$**0.05**
	Gompertz	-10071.5	20157.1	-
	Weibull	-10288.8	20591.7	-
	Generalized Gamma	-9797.5	19611.2	-
	Log-normal	-9994.2	20002.5	-
4	Cox-PH	-7549.5	15100.1	$<$**0.01**
	Gompertz	-7155.4	14324.9	-
	Weibull	-7160.7	14335.6	-
	Generalized Gamma	-7067.8	14151.7	-
	Log-normal	-7082.7	14179.4	-

*Proportional hazard assumption diagnostics using cox.zph

TABLE II

A COMPARISON OF HAZARD RATIO AND PARAMETER ESTIMATION WITH CONFIDENCE INTERVAL

Transition	Variable	Cause-Specific				Cumulative Incidence Function			
		Parametric		Semi-Parametric		Parametric		Semi-Parametric	
		e^{β}	(95 %CI)	HR	(95 %CI)	e^{β}	(95 %CI)	HR	(95 %CI)
1 (Non-competing)	Off-ramp (vs.w/o)	**1.42**	(1.18,1.71)	**0.61**	(0.47,0.80)	**1.42**	(1.18,1.71)	**0.61**	(0.47,0.80)
	Car (vs. truck)	**1.28**	(1.03,1.59)	**0.60**	(0.43,0.83)	**1.28**	(1.03,1.59)	**0.60**	(0.43,0.83)
	Above speed limit (vs. below)	0.98	(0.82,1.19)	1.13	(0.86,1.48)	0.98	(0.82,1.19)	1.13	(0.86,1.48)
	Acceleration (vs. deceleration)	1.12	(0.89,1.40)	0.93	(0.66,1.30)	1.12	(0.89,1.40)	0.93	(0.66,1.30)
	Adjacent (vs. non-adjacent)	**1.33**	(1.12,1.58)	**0.64**	(0.49,0.83)	**1.33**	(1.12,1.58)	**0.64**	(0.49,0.83)
2 (Competing)	Off-ramp (vs. w/o)	0.91	(0.72,1.16)	1.04	(0.77,1.41)	0.90	(0.69, 1.17)	1.25	(0.92, 1.70)
	Car (vs. truck)	**0.65**	(0.49,0.86)	**2.21**	(1.47,3.32)	0.98	(0.55, 1.71)	**2.44**	(1.63, 3.65)
	Above speed limit (vs. below)	0.90	(0.72,1.13)	1.30	(1.00,1.68)	1.02	(0.82, 1.26)	1.20	(0.93, 1.56)
	Acceleration (vs. deceleration)	0.88	(0.69,1.13)	1.27	(0.92,1.76)	0.92	(0.76, 1.11)	**1.44**	(1.05, 1.98)
	Adjacent (vs. non-adjacent)	**1.44**	(1.16,1.79)	**0.58**	(0.45,0.75)	0.97	(0.73, 1.27)	0.91	(0.72, 1.17)
3 (Competing)	Off-ramp (vs. w/o)	**1.28**	(1.16,1.42)	**0.78**	(0.68,0.88)	**1.27**	(1.14, 1.41)	**0.79**	(0.70, 0.89)
	Car (vs. truck)	1.10	(1.00,1.21))	**0.82**	(0.73,0.93)	1.02	(0.92, 1.12)	**0.77**	(0.68, 0.87)
	Above speed limit (vs. below)	1.00	(0.92,1.09)	1.08	(0.97, 1.21)	1.08	(0.99, 1.19)	0.96	(0.85, 1.07)
	Acceleration (vs. deceleration)	**1.17**	(1.06,1.30)	**0.84**	(0.73, 0.96)	**1.15**	(1.03, 1.28)	**0.82**	(0.72, 0.93)
	Adjacent (vs. non-adjacent)	**1.56**	(1.45,1.68)	**0.58**	(0.52,0.64)	**1.35**	(1.24, 1.47)	**0.75**	(0.68, 0.83)
4 (Non-competing)	Off-ramp (vs. w/o)	0.95	(0.81,1.12)	0.99	(0.83,1.19)	0.95	(0.81,1.12)	0.99	(0.83,1.19)
	Car (vs. truck)	**0.84**	(0.72,0.98)	1.17	(0.99, 1.37)	**0.84**	(0.72,0.98)	1.17	(0.99, 1.37)
	Above speed limit (vs. below)	**1.22**	(1.06,1.40)	**0.73**	(0.63, 0.84)	**1.22**	(1.06,1.40)	**0.73**	(0.63, 0.84)
	Acceleration (vs. deceleration)	**1.22**	(1.03,1.43)	**0.79**	(0.66, 0.95)	**1.22**	(1.03,1.43)	**0.79**	(0.66, 0.95)
	Adjacent (vs. non-adjacent)	**2.33**	(2.06,2.63)	**0.41**	(0.36, 0.47)	**2.33**	(2.06,2.63)	**0.41**	(0.36, 0.47)

events. Moreover, the acceleration and presence of nearby vehicles can both increase the de-escalation duration. The results suggest the underlying impact of the off-ramp on traffic safety hazards before accidents occur.

One limitation of the cause-specific approach is that it can be a biased estimator for predicting marginal survival functions as shown in Fig. 4. The cumulative function is inflated and it crossed the other events' survival function. Due to treating the competing events as censored, it can have an upward bias for the cumulative function, since $I_{2,1}(x) = 1 - \int_0^x \lambda_{2,1}(u) S_2(u) du < 1 - \int_0^x \lambda_{2,1}(u) S_{21}(u) du$, and a downward bias for the marginal survival function, since $S_{2,k}(x) < 1 - I_{2,j} = e^{-H_{2,j}(x)}$ [13, 34]. As a comparison, the CIF approach treating the competing risks as potential risk sets can adjust the biases, and provide a better understanding than using the cause-specific hazard approach alone [35].

C. Model Prediction

We further applied CIF to predict marginal distribution functions. For de-escalation transition 3, the presence of an off-ramp has increased the subdistribution hazard rate by a factor of 1.27 (95% CI: 1.14-1.41), which is significant at 0.05 level. Compared with trucks, vehicles on average have increased the hazard by a factor of 1.02 (95% CI 0.92-1.12), however, the impacts of vehicle types are found to be insignificant. Fig. 5 predicts the marginal survival probability of transition 3 and the cumulative function of transition 2 for interaction pairs at the risky states. The presence of an off-ramp has increased the median durations of a vehicle transitioning from a risky state to a safe state by 27% compared with one without an off-ramp.

One limitation is that our naturalistic driving data is non-experimental data, and its covariate effects should not be interpreted as a causal effect, which requires a randomized control experiment or causation analysis [1]. Nevertheless,

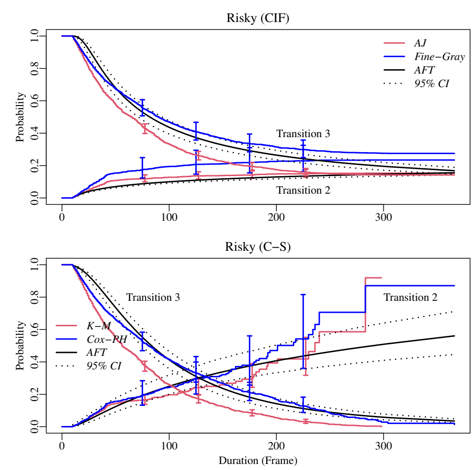

Fig. 4. A comparison of survival curves and cumulative incidence function curve between cause-specific and cumulative incidence approach with 95% confidence intervals. The two competing event curves crossed suggesting the cause-specific has a biased estimation for marginal survival function.

the model results can help to understand the correlation between risk-driving behaviors and off-ramps under the context of multi-state state transitions.

V. CONCLUSIONS

We used Markov-renewal multi-state survival models to analyze highway naturalistic driving data from Germany. Best-fitted parametric distributions for C-S de-escalation transitions are log-normal, and generalized gamma distributions for escalation transitions. Results show the presence of nearby vehicles can increase the duration of both escalation and de-escalation transitions significantly. Adjusted for other variables, off-ramp can statistically reduce the cause-specific hazard of de-escalation transitions by a factor of $1.16 - 1.42$ at risky states. Its impact on escalation transitions, however,

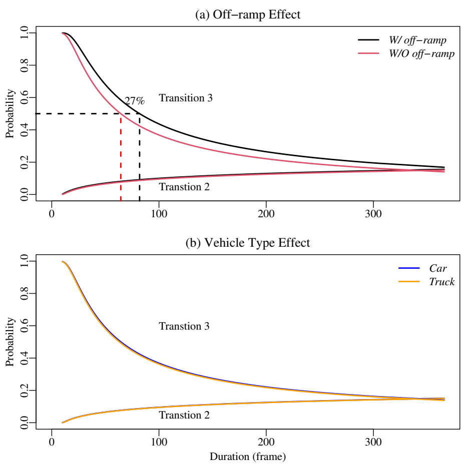

Fig. 5. The effects of off-ramp and vehicle types on escalation and de-escalation transitions from risky states with average speed above the speed limit, average acceleration greater than 0, and presence of nearby vehicles. The CIF approach is recommended for predicting marginal distributions.

is not significant. Cars show a $14\% - 51\%$ reduction in the escalation transition duration compared with trucks significantly. Although the cause-specific approach has its benefit in interpretation, we demonstrated its limitation in predicting marginal survival functions as shown in Fig. 4. Hence, we recommend using the cumulative incidence approach as a supplement when competing events are present in highway naturalistic driving data.

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